

# ENE4014: Programming Languages

## Lecture 8 — Design and Implementation of PLs (4) States

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# Review: Our Language So Far

Our language has expressions and procedures.

Syntax

$$\begin{array}{l} P \rightarrow E \\ E \rightarrow n \\ \quad | x \\ \quad | E + E \\ \quad | E - E \\ \quad | \text{iszero } E \\ \quad | \text{if } E \text{ then } E \text{ else } E \\ \quad | \text{let } x = E \text{ in } E \\ \quad | \text{read} \\ \quad | \text{letrec } f(x) = E \text{ in } E \\ \quad | \text{proc } x E \\ \quad | E E \end{array}$$

# Review: Our Language So Far

## Semantics

$$\frac{}{\rho \vdash n \Rightarrow n} \quad \frac{}{\rho \vdash x \Rightarrow \rho(x)} \quad \frac{\rho \vdash E_1 \Rightarrow n_1 \quad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \text{iszero } E \Rightarrow \text{true}} \quad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \text{iszero } E \Rightarrow \text{false}} \quad n \neq 0 \quad \frac{}{\rho \vdash \text{read} \Rightarrow n}$$

$$\frac{\rho \vdash E_1 \Rightarrow \text{true} \quad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v} \quad \frac{\rho \vdash E_1 \Rightarrow \text{false} \quad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v} \quad \frac{[f \mapsto (f, x, E_1, \rho)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow v}$$

$$\frac{}{\rho \vdash \text{proc } x \ E \Rightarrow (x, E, \rho)}$$

$$\frac{\rho \vdash E_1 \Rightarrow (x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 \ E_2 \Rightarrow v'}$$

$$\frac{\rho \vdash E_1 \Rightarrow (f, x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v, f \mapsto (f, x, E, \rho')]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 \ E_2 \Rightarrow v'}$$

# This Lecture: Adding States to the Language

- So far, our language only had the values produced by computation.
- But computation also has *effects*: it may change the state of memory.
- We will extend the language to support computational effects:
  - ▶ Syntax for creating and using memory locations
  - ▶ Semantics for manipulating memory states

## Motivating Example

- How can we compute the number of times `f` has been called?

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let f = proc (x) (x)
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let counter = 0
in let f = proc (x) (let counter = counter + 1
                    in x)
   in let a = (f (f 1))
      in counter
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- Does the following program work?

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let counter = 0
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   in let a = (f (f 1))
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```

- The binding of `counter` is local. We need global *effects*.
- Effects are implemented by introducing *memory (store)* and *locations (reference)*.

## Two Approaches

Programming languages support references explicitly or implicitly.

- Languages with explicit references provide a clear account of allocation, dereference, and mutation of memory cells.
  - ▶ e.g., OCaml, F#
- In languages with implicit references, references are built-in. References are not explicitly manipulated.
  - ▶ e.g., C and Java.



# A Language with Explicit References

$$\begin{array}{l} P \rightarrow E \\ E \rightarrow n \mid x \\ \quad | E + E \mid E - E \\ \quad | \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ \quad | \text{let } x = E \text{ in } E \\ \quad | \text{proc } x E \mid E E \\ \quad | \text{ref } E \\ \quad | ! E \\ \quad | E := E \\ \quad | E; E \end{array}$$

- $\text{ref } E$  allocates a new location, store the value of  $E$  in it, and returns it.
- $! E$  returns the contents of the location that  $E$  refers to.
- $E_1 := E_2$  changes the contents of the location ( $E_1$ ) by the value of  $E_2$ .
- $E_1; E_2$  executes  $E_1$  and then  $E_2$  while accumulating effects.

## Example 1

- ```
let counter = ref 0
in let f = proc (x) (counter := !counter + 1; !counter)
    in let a = (f 0)
        in let b = (f 0)
            in (a - b)
```

## Example 1

- ```
let counter = ref 0
  in let f = proc (x) (counter := !counter + 1; !counter)
      in let a = (f 0)
          in let b = (f 0)
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```
- ```
let f = let counter = ref 0
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      in let b = (f 0)
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## Example 1

- `let counter = ref 0`  
  `in let f = proc (x) (counter := !counter + 1; !counter)`  
    `in let a = (f 0)`  
      `in let b = (f 0)`  
        `in (a - b)`
- `let f = let counter = ref 0`  
      `in proc (x) (counter := !counter + 1; !counter)`  
`in let a = (f 0)`  
   `in let b = (f 0)`  
      `in (a - b)`
- `let f = proc (x) (let counter = ref 0`  
                  `in (counter := !counter + 1; !counter))`  
`in let a = (f 0)`  
   `in let b = (f 0)`  
      `in (a - b)`

## Example 2

We can make chains of references:

```
let x = ref (ref 0)
in (!x := 11; !(!x))
```

# Semantics

Memory is modeled as a finite map from locations to values:

$$\begin{aligned} \mathit{Val} &= \mathbb{Z} + \mathit{Bool} + \mathit{Procedure} + \mathit{Loc} \\ \mathit{Procedure} &= \mathit{Var} \times \mathit{E} \times \mathit{Env} \\ \rho \in \mathit{Env} &= \mathit{Var} \rightarrow \mathit{Val} \\ \sigma \in \mathit{Mem} &= \mathit{Loc} \rightarrow \mathit{Val} \end{aligned}$$

Semantics rules additionally describe memory effects:

$$\rho, \sigma \vdash E \Rightarrow v, \sigma'$$

# Semantics

Existing rules are enriched with memory effects:

$$\frac{}{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \frac{}{\rho, \sigma \vdash x \Rightarrow \rho(x), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow n, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{false}, \sigma_1} \quad n \neq 0$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{true}, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{false}, \sigma_1 \quad \rho, \sigma_1 \vdash E_3 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad [x \mapsto v_1]\rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v, \sigma_2}$$

$$\frac{}{\rho, \sigma \vdash \text{proc } x \ E \Rightarrow (x, E, \rho), \sigma}$$

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# Semantics

Rules for new constructs:

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \text{ref } E \Rightarrow l, [l \mapsto v]\sigma_1} \quad l \notin \text{Dom}(\sigma_1)$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow l, \sigma_1}{\rho, \sigma_0 \vdash ! E \Rightarrow \sigma_1(l), \sigma_1}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow l, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash E_1 := E_2 \Rightarrow v, [l \mapsto v]\sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v_2, \sigma_2}{\rho, \sigma_0 \vdash E_1; E_2 \Rightarrow v_2, \sigma_2}$$



## Example

---

$$\rho, \sigma_0 \vdash \text{let } x = \text{ref } (\text{ref } 0) \text{ in } (!x := 11; !(!x)) \Rightarrow$$

## Exercise

Extend the language with recursive procedures:

$$\begin{aligned} P &\rightarrow E \\ E &\rightarrow n \mid x \\ &\mid E + E \mid E - E \\ &\mid \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ &\mid \text{let } x = E \text{ in } E \\ &\mid \text{letrec } f(x) = E \text{ in } E \\ &\mid \text{proc } x E \mid E E \\ &\mid \text{ref } E \\ &\mid ! E \\ &\mid E := E \\ &\mid E; E \end{aligned}$$

## Exercise (Continued)

- Domain:

$$\begin{aligned} \mathit{Val} &= \mathbb{Z} + \mathit{Bool} + \mathit{Procedure} + \mathit{Loc} \\ \mathit{Procedure} &= \mathit{Var} \times \mathit{E} \times \mathit{Env} \\ \rho \in \mathit{Env} &= \mathit{Var} \rightarrow \mathit{Val} \\ \sigma \in \mathit{Mem} &= \mathit{Loc} \rightarrow \mathit{Val} \end{aligned}$$

- Semantics rules:

$$\frac{}{\rho, \sigma_0 \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow}$$

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# A Language with Implicit References

$$\begin{aligned} P &\rightarrow E \\ E &\rightarrow n \mid x \\ &\mid E + E \mid E - E \\ &\mid \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ &\mid \text{let } x = E \text{ in } E \\ &\mid \text{proc } x E \mid E E \\ &\mid x := E \\ &\mid E; E \end{aligned}$$

- In this design, every variable denotes a reference and is mutable.
- $x := E$  changes the contents of  $x$  by the value of  $E$ .

## Examples

Computing the number of times `f` has been called:

- ```
let counter = 0
  in let f = proc (x) (counter := counter + 1; counter)
      in let a = (f 0)
          in let b = (f 0)
              in (a-b)
```

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  `in let b = (f 0)`  
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- `let f = proc (x) (let counter = 0`  
  `in (counter := counter + 1; counter))`  
`in let a = (f 0)`  
  `in let b = (f 0)`  
    `in (a-b)`

## Exercise

What is the result of the program?

```
let f = proc (x)
          proc (y)
            (x := x + 1; x - y)
        in ((f 44) 33)
```



# Semantics

References are no longer values and every variable denotes a reference:

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# Semantics

$$\overline{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \overline{\rho, \sigma \vdash x \Rightarrow \sigma(\rho(x)), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1}$$

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## Example

```
let f = let count = 0
        in proc (x) (count := count + 1; count)
in let a = (f 0)
    in let b = (f 0)
        in a - b
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## Exercise (Continued)

- Domain:

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- Semantics rules:

$$\frac{}{\rho, \sigma_0 \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow}$$

$$\frac{}{\rho, \sigma_0 \vdash E_1 E_2 \Rightarrow}$$

## Parameter-Passing Variations

- Our current strategy of calling a procedure is *call-by-value*. The formal parameter refers to a new location containing the value of the actual parameter:

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \quad [x \mapsto l]\rho', [l \mapsto v]\sigma_2 \vdash E \Rightarrow v', \sigma_3}{\rho, \sigma_0 \vdash E_1 E_2 \Rightarrow v', \sigma_3} \quad l \notin \text{Dom}(\sigma_2)$$

- The most commonly used form of parameter-passing.

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- The most commonly used form of parameter-passing.
- For example, the assignment to `x` has no effect on the contents of `a`:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```

## Parameter-Passing Variations

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- The most commonly used form of parameter-passing.
- For example, the assignment to `x` has no effect on the contents of `a`:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```

- Under *call-by-reference*, the assignment changes the value of `a` after the call.



# Call-By-Reference Parameter-Passing

The location of the caller's variable is passed, rather than the contents of the variable.

- Extend the syntax:

$$\begin{array}{l} E \rightarrow \vdots \\ | \quad E \ E \\ | \quad E \langle y \rangle \end{array}$$

- Extend the semantics:

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad [x \mapsto \rho(y)]\rho', \sigma_1 \vdash E \Rightarrow v', \sigma_2}{\rho, \sigma_0 \vdash E_1 \langle y \rangle \Rightarrow v', \sigma_2}$$

What is the benefit of call-by-reference compared to call-by-value?

## Examples

- `let p = proc (x) (x := 4)`  
  `in let a = 3`  
    `in ((p <a>); a)`

## Examples

- `let p = proc (x) (x := 4)`  
  `in let a = 3`  
    `in ((p <a>); a)`
- `let f = proc (x) (x := 44)`  
  `in let g = proc (y) (f <y>)`  
    `in let z = 55`  
      `in ((g <z>); z)`

## Examples

- `let p = proc (x) (x := 4)`  
  `in let a = 3`  
    `in ((p <a>); a)`
- `let f = proc (x) (x := 44)`  
  `in let g = proc (y) (f <y>)`  
    `in let z = 55`  
      `in ((g <z>); z)`
- `let swap = proc (x) proc (y)`  
    `let temp = x`  
    `in (x := y; set y = temp)`  
`in let a = 33`  
  `in let b = 44`  
    `in (((swap <a>) <b>); (a-b))`

## Variable Aliasing

More than one call-by-reference parameter may refer to the same location:

```
let b = 3
in let p = proc (x) proc (y)
      (x := 4; y)
  in ((p <b>) <b>)
```

- A *variable aliasing* is created:  $x$  and  $y$  refer to the same location
- With aliasing, reasoning about program behavior is very difficult, because an assignment to one variable may change the value of another.

# Lazy Evaluation

- So far all the parameter-passing strategies are *eager* in that they always evaluate the actual parameter before calling a procedure.
- In eager evaluation, procedure arguments are completely evaluated before passing them to the procedure.
- On the other hand, *lazy evaluation* delays the evaluation of arguments until it is actually needed. If the procedure body never uses the parameter, it will never be evaluated.
- Lazy evaluation potentially avoids non-termination:

```
letrec infinite(x) = (infinite x)
in let f = proc (x) (1)
   in (f (infinite 0))
```

# Lazy Evaluation

## Comparison to Eager evaluation

Eager Evaluation	Lazy Evaluation
Space-efficient	Time-efficient
Cannot avoid non-termination	Can avoid non-termination
Easy to reason with the order of evaluation	Hard
Easy to reason with programs with effects	Hard

# Summary

Our language is now (somewhat) realistic:

- expressions, procedures, recursion,
- states with explicit/implicit references
- parameter-passing variations