ENE4014: Programming Languages

Lecture 12 — Type System
(3) Manual Type Annotation

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Typing Rules

$$\frac{}{\Gamma \vdash n : \text{int}} \qquad \frac{}{\Gamma \vdash x : \Gamma(x)}$$

$$\frac{\Gamma \vdash E_1 : \text{int} \qquad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 + E_2 : \text{int}} \qquad \frac{\Gamma \vdash E_1 : \text{int} \qquad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 - E_2 : \text{int}}$$

$$\frac{\Gamma \vdash E : \text{int}}{\Gamma \vdash \texttt{iszero}\ E : \text{bool}} \qquad \frac{\Gamma \vdash E_1 : \text{bool} \qquad \Gamma \vdash E_2 : t \qquad \Gamma \vdash E_3 : t}{\Gamma \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 : t}$$

$$\frac{\Gamma \vdash E_1 : t_1 \qquad [x \mapsto t_1]\Gamma \vdash E_2 : t_2}{\Gamma \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 : t_2} \qquad \frac{\Gamma \vdash E_1 : t_1 \to t_2 \qquad \Gamma \vdash E_2 : t_1}{\Gamma \vdash E_1\ E_2 : t_2}$$

$$\frac{[x \mapsto t_1]\Gamma \vdash E : t_2}{\Gamma \vdash \texttt{proc}\ x\ E : t_1 \to t_2}$$

Implementation: First Try

Can we implement the type checker by recursively (like interpreter)?

let rec **typeof** $\Gamma\ E =$
  match E with
  | $n \rightarrow$ int
  | $x \rightarrow \Gamma(x)$
  | $E_1 + E_2 \rightarrow$
    let $t_1 =$ **typeof** $\Gamma\ E_1$
    let $t_2 =$ **typeof** $\Gamma\ E_2$
      if $t_1 =$ int and $t_2 =$ int then int
      else raise TypeError
      $\vdots$

Challenge

Given a program $\boldsymbol{E}$, how to check $[] \vdash \boldsymbol{E} : \boldsymbol{t}$? Nontrivial, because of the following type rule:

$$\frac{[\boldsymbol{x} \mapsto \boldsymbol{t_1}]\boldsymbol{\Gamma} \vdash \boldsymbol{E} : \boldsymbol{t_2}}{\boldsymbol{\Gamma} \vdash \texttt{proc}\ \boldsymbol{x}\ \boldsymbol{E} : \boldsymbol{t_1} \rightarrow \boldsymbol{t_2}}$$

Two approaches:

- *Type Annotation*: Programmers are required to supply the type of the function argument. Used in C, C++, Java, etc.
- *Type Inference*: Type checker attempts to automatically infer types. Only possible if the language is carefully designed. Used in ML, Haskell, etc.

Language with Type Annotation

Consider the language with (recursive) procedures:

$$
\begin{array}{rcl}
E & \to & n \\
& | & x \\
& | & E + E \\
& | & E - E \\
& | & \texttt{iszero } E \\
& | & \texttt{if } E \texttt{ then } E \texttt{ else } E \\
& | & \texttt{let } x = E \texttt{ in } E \\
& | & \texttt{proc } (x : t)\ E \\
& | & \texttt{letrec } t_1\ f(x : t_2) = E \texttt{ in } E \\
& | & E\ E
\end{array}
$$

Typing Rules

$$\frac{}{\Gamma \vdash n : \text{int}} \qquad \frac{}{\Gamma \vdash x : \Gamma(x)}$$

$$\frac{\Gamma \vdash E_1 : \text{int} \qquad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 + E_2 : \text{int}} \qquad \frac{\Gamma \vdash E_1 : \text{int} \qquad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 - E_2 : \text{int}}$$

$$\frac{\Gamma \vdash E : \text{int}}{\Gamma \vdash \texttt{iszero}\ E : \text{bool}} \qquad \frac{\Gamma \vdash E_1 : \text{bool} \qquad \Gamma \vdash E_2 : t \qquad \Gamma \vdash E_3 : t}{\texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 : t}$$

$$\frac{\Gamma \vdash E_1 : t_1 \qquad [x \mapsto t_1]\Gamma \vdash E_2 : t_2}{\Gamma \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 : t_2} \qquad \frac{\Gamma \vdash E_1 : t_1 \to t_2 \qquad \Gamma \vdash E_2 : t_1}{\Gamma \vdash E_1\ E_2 : t_2}$$

$$\frac{[x \mapsto t_1]\Gamma \vdash E : t_2}{\Gamma \vdash \texttt{proc}\ (x : t_1)\ E : t_1 \to t_2} \qquad \frac{\begin{array}{c}[x \mapsto t_2, f \mapsto (t_2 \to t_1)]\Gamma \vdash E_1 : t_1 \\ [f \mapsto (t_2 \to t_1)]\Gamma \vdash E_2 : t\end{array}}{\Gamma \vdash \texttt{letrec}\ t_1\ f(x : t_2) = E_1\ \texttt{in}\ E_2 : t}$$

Example 1

$$\frac{}{[] \vdash \texttt{proc}\ (x : \mathsf{int})\ (x + 1) :}$$

Example 2

$$[] \vdash \texttt{letrec int } \boldsymbol{dbl}\ (\boldsymbol{x} : \text{int}) = \texttt{if iszero } \boldsymbol{x} \texttt{ then } \mathbf{0}$$
$$\texttt{else } (\boldsymbol{dbl}\ (\boldsymbol{x} - \mathbf{1})) + \mathbf{2} \texttt{ in } \boldsymbol{dbl}\ \mathbf{2} :$$

Example 3

$$\frac{}{[] \vdash \texttt{proc}\ (f : (\text{bool} \to \text{int}))\ \texttt{proc}\ (n : \text{int})\ (f\ (\texttt{iszero}\ n))}$$

$$\vdots$$

Type Check Algorithm

Now we can implement the type checking algorithm recursively:

let rec **typeof** $\Gamma\ E =$
  match E with
  | `proc` $(x : t_1)\ E_1 \rightarrow$
    $\vdots$