Homework 3
ENE4014 Programming Languages, Spring 2024
due: 5/6(Mon), 23:59

- You must write your code by yourself and must not look at someone elses code.
- Do not use any external libraries. You can use only the OCaml standard library (`https://ocaml.org/api/index.html`).
- Exercises 12 – 15 are optional. If you want to get extra points, you can solve them.

Exercise 1 (5 points) Write a function

```
revrev:  'a list list -> 'a list list
```

such that `revrev`t returns the result of reversing the order of the elements of t and then reversing the order of the elements of each element of t. For example,

```
revrev [[1;2;3]; [4;5;6]; [7;8;9]] = [[9;8;7]; [6;5;4]; [3;2;1]]
```

□

Exercise 2 (5 points) Write a function

```
union:  'a list -> 'a list -> 'a list
```

such that `union` t_1 t_2 returns the union of the elements of t_1 and t_2 without duplicates. You can use '=' to compare elements of the list. The order of elements in the return value does not matter. For example,

$$\texttt{union}\ [1;2;3;4]\ [3;4;5;6] = [1;2;3;4;5;6].$$

□

Exercise 3 (5 points) Write a function

```
alterSum:  int list -> int
```

such that `alterSum` t returns returns an integer that is the result of applying addition and subtraction to the elements of the list t alternately. The first operation, if applicable, is addition. For an empty list, `alterSum` returns 0. For example,

$$\texttt{alterSum}\ [] = 0$$

$$\texttt{alterSum}\ [1;2;3;4;5] = 1+2-3+4-5 = -1$$

$$\texttt{alterSum}\ [1;2;3;4;5;6] = 1+2-3+4-5+6 = 5$$

□

Exercise 4 (5 points) Write a function

```
dsort:  int list -> int list
```

such that `dsort` t returns a list that is the result of sorting the elements of t in descending order. For example,

$$\texttt{dsort}\ [3;2;1;4;5] = [5;4;3;2;1]$$

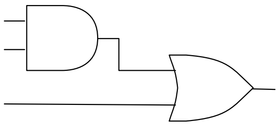
$$\texttt{dsort}\ [2;1;4;3;6;5] = [6;5;4;3;2;1].$$

Do not use any built-in sorting functions (e.g., List.sort, List.stable_sort, List.fast_sort, etc.). □

Exercise 5 Consider the following OCaml data type for Boolean circuits

```
type circuit = IN
  | AND of circuit * circuit
  | OR of circuit * circuit
```

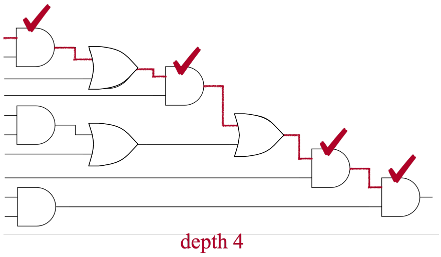
where `IN` denotes input. For example, the following circuit



can be described as

```
OR (AND (IN, IN), IN)
```

The AND depth of a circuit is the maximum number of sequential AND gates from input to output. For example, the following circuit has the AND depth of 4.



Write a function

$$\texttt{and\_depth : circuit -> int}$$

that takes a circuit and returns its AND depth.

Exercise 6 (5 points) Write a higher-order function

$$\texttt{iter}\ n\ f = f^n$$

where f^n is the function that is the result of applying f to itself n times (in other words, $f^n(x) = \underbrace{f(\cdots(f}_{n}(x))$. When $n = 0$, the function returns the identity function (`fun x -> x`).

For example,

$$\texttt{(iter}\ n\ f\texttt{) 0}$$

when $f =$ `(fun x -> x + 2)` and $n = 3$ returns 6 because

$$\texttt{(iter}\ n\ f\texttt{) 0} = f^3(0) = f(f(f(0))) = f(f(2)) = f(4) = 6.$$

□

Exercise 7 (5 points) Write a function

```
mapn:  ('a -> 'a) -> int -> 'a list -> 'a list
```

such that `mapn` f n l returns a list that is the result of applying f to each element of l n times. More precisely,

$$\texttt{mapn}\ f\ n\ [x_1; x_2; \cdots, x_m] = [f^n(x_1); f^n(x_2); \cdots, f^n(x_m)]$$

where f^n is defined as in the previous exercise and x_i is the i-th element of l. For example,

```
mapn (fun x -> x + 1) 3 [1; 2; 3; 4; 5] = [4; 5; 6; 7; 8]

mapn (fun x -> x + 1) 0 [1; 2; 3; 4; 5] = [1; 2; 3; 4; 5]
```

□

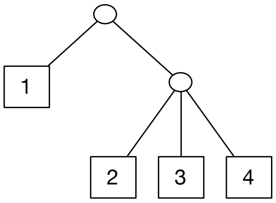
Exercise 8 (5 points) N-ary tree is a tree in which each node has at most N children. The following is the definition of N-ary tree whose elements are of type `'a`.

```
type 'a ntree = Leaf of 'a | Node of ('a ntree list)
```

For example,

```
Node [Leaf 1; Node [Leaf 2; Leaf 3; Leaf 4]]
```

is a tree represented as follows:



Write a function

```
findn:  'a ntree -> int
```

such that `findn` t returns the maximum number of children of any node in the tree t. For example,

```
findn (Node [Leaf 1; Node [Leaf 2; Leaf 3; Leaf 4]]) = 3
```

□

Exercise 9 Write a function

```
flatten:  'a ntree -> 'a list
```

such that `flatten` t returns a list containing all the `'a` type-elements in the tree t. The order of elements in the return value does not matter. For example,

```
flatten (Node [Leaf 1; Node [Leaf 2; Leaf 2; Leaf 4]]) = [1; 2; 2; 4]
```

□

Exercise 10 (5 points) Consider the following OCaml data type for propositional formulas

```
type formula = TRUE | FALSE
  | NOT of formula
  | ANDALSO of formula * formula
  | ORELSE of formula * formula
  | IMPLY of formula * formula
  | LESS of expr * expr
and expr = NUM of int
  | PLUS of expr * expr
  | MINUS of expr * expr
```

Considering the above definition, write a function

$$\texttt{eval} : \texttt{formula} \rightarrow \texttt{bool}$$

that computes the truth value of a given formula. For example,

```
eval (IMPLY (IMPLY (TRUE, FALSE), TRUE))
```

evaluates to *true*, and

```
eval (LESS (NUM 5, PLUS (NUM 1, NUM 2)))
```

evaluates to *false*.

Exercise 11 (5 points) Write a function

```
sigma :  int * int * (int -> int) -> int.
```

such that `sigma(a,b,f)` returns $\Sigma_{n=a}^{b} f(n)$.

Exercise 12 (**Optional**, 10 points) Write a function

$$\texttt{diff} : \texttt{ae} * \texttt{string} \rightarrow \texttt{ae}$$

that differentiates the given algebraic expression with respect to the variable given as the second argument. The `ae` type is defined as follows:

```
type ae = CONST of int
  | VAR of string
  | POWER of string * int
  | TIMES of ae list
  | SUM of ae list
```

For example, $x^2 + 2x + 1$ is represented by

```
SUM [POWER ("x", 2); TIMES [CONST 2; VAR "x"]; CONST 1]
```

and differentiating it (w.r.t. "x") gives $2x + 2$, which can be represented by

```
SUM [TIMES [CONST 2; VAR "x"]; CONST 2]
```

Exercise 13 (**Optional**, 10 points) Write a function

$$\texttt{calculate} : \texttt{exp} \rightarrow \texttt{float}$$

that returns a result of a given arithmetic formula. The `exp` type is defined as follows:

```
  type exp = X | INT of int
    | REAL of float
    | ADD of exp * exp
    | SUB of exp * exp
    | MUL of exp * exp
    | DIV of exp * exp
    | SIGMA of exp * exp * exp
    | INTEGRAL of exp * exp * exp
```

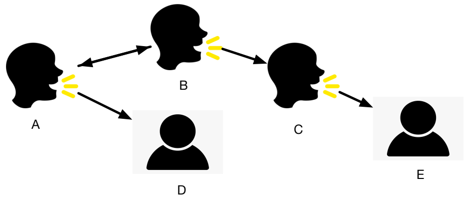
For example, the following arithmetic formulas can be written in the exp type:

$$\sum_{x=1}^{10}(x \times x - 1) \qquad \mathtt{SIGMA(INT1, INT10, SUB(MUL(X, X), INT1))}$$

$$\int_{x=1.0}^{10.0}(x \times x - 1)dx \qquad \mathtt{INTEGRAL(REAL1.0, REAL10.0, SUB(MUL(X, X), INT1))}$$

When you compute integrals, dx should be 0.1.

Exercise 14 (**Optional**, 10 points) Suppose we are interested in if someone gets COVID-19. The following image describes people who are talking to someone else in a party at a moment.



The type `talkingto` is for representing whom is each person talking to:

```
  type talkingto = (string * string) list
```

For example, the above situation can be represented as

```
let party = [("A", "B"); ("B", "A"); ("A", "D"); ("B", "C"); ("C", "E")]
```

Suppose no one is wearing a mask, and if person A talks to B and A gets COVID-19, B also gets infected immediately. For example, in the above situation, E gets infected if A got COVID because A talks to B, who talks to C, who talks to E.

Write a function

```
        infected :  talkingto -> string -> string -> bool
```

that determines if a person (3rd argument) gets infected when another person (2nd argument) got COVID. For example, the function should behave as follows:

```
  infected party "A" "E" = true
  infected party "B" "D" = true
  infected party "C" "D" = false
  infected party "C" "B" = false
```

Exercise 15 (**Optional**, 10 points) As an extension of the previous exercise, suppose people who get vaccinated never get COVID. Write a function

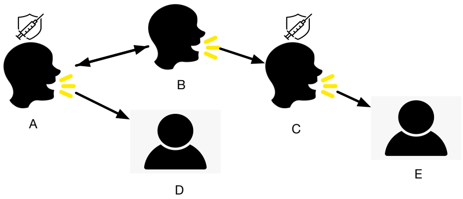
```
infected_vaccine :  talkingto -> string list -> string -> string -> bool
```

that additionally gets a list of people who get vaccinated as the second argument.

For example,

```
  infected_vaccine party ["A"; "C"] "B" "D" = false
```

as if A and C get vaccinated, E is free from COVID even if B got infected because C is blocking the way from B to E.



The followings are other example behaviors.

```
  infected_vaccine party ["A"; "C"] "B" "E" = false
  infected_vaccine party ["A"] "B" "E" = true
  infected_vaccine party ["C"] "A" "E" = false
```