Homework 1
ENE4014 Programming Languages, Spring 2024
due: 4/01 (Mon)

- Submit your answer (as a hard copy) to the instructor after the class on 4/1 (do not forget to write your name and student ID!).

Exercise 1. (3 points) Write inductive definitions of the following set

$$S = \{3n + 2 \mid n \in \mathbb{N}\}$$

in all three styles (top-down, bottom-up, and rules of inference).

Exercise 2. (3 points) Write inductive definitions of the following set

$$S = \{2n + 3m + 1 \mid n, m \in \mathbb{N}\}$$

in all three styles (top-down, bottom-up, and rules of inference).

Exercise 3. (4 points) What sets of strings are defined by the following rules?

1. $\frac{}{\texttt{a}} \quad \frac{}{\texttt{b}} \quad \frac{x_1 \quad x_2}{x_1 \texttt{c} x_2}$
2. $\frac{}{(\,)} \quad \frac{x}{((x))}$

Exercise 4. (2 points) Consider a set of binary trees inductively defined as follows:

$$\begin{array}{rcl} t & \to & n \quad (n \in \mathbb{N}) \\ & | & (n, t, t) \end{array}$$

Write a derivation tree for the binary tree $(5, 4, (1, 2, 3))$.

Exercise 5. (4 points) Consider a set of lists inductively defined as follows:

$$\begin{array}{rcl} t & \to & \texttt{nil} \\ & | & n \cdot m \cdot t \quad (n, m \in \mathbb{N}) \end{array}$$

Prove every list contains an even number of integers.

More formally, let $i(t)$ denote the number of integers that appear in list t. Our goal is to prove the following property over every $t \in T$:

$$i(t) = 2k \text{ for some } k \in \mathbb{N}$$

Exercise 6. (4 points) Consider the set of binary trees in Exercise 4. Prove every tree contains an odd number of integers.

More formally, let $i(t)$ denote the number of integers that appear in tree t. Our goal is to prove the following property over every $t \in T$:

$$i(t) = 2k + 1 \text{ for some } k \in \mathbb{N}.$$