Final Exam
ENE4014 Programming Languages, Spring 2022
6/13 (Mon), 10:30

Problem 1 [Rules of inferences] (20 pts)

a) (4 pts) Complete the following inference rules for binary trees. Examples of binary trees include 1, $(1, nil)$, $(1,2)$, $(nil,(1,2))$, $((1,2),(3,4))$, etc. The inference rules are:

$$\frac{}{n}\; n \in \mathbb{Z} \qquad \frac{t}{(t, nil)} \qquad \frac{t}{\boxed{\text{(nil, t)}}} \qquad \frac{t_1 \quad t_2}{\boxed{(\mathrm{t}_1, t_2)}}$$

b) (8 pts) Consider the following grammar for a set S of Boolean formulas.

$$f \to T \mid F \mid f \wedge f \mid f \vee f$$

Complete the following derivation tree for proving that $(T \wedge F) \vee (F \vee T)$ is in S:

$$\frac{\dfrac{\dfrac{}{T} \quad \dfrac{}{F}}{(T \wedge F)} \qquad \dfrac{\dfrac{}{F} \quad \dfrac{}{T}}{(F \vee T)}}{(T \wedge F) \vee (F \vee T)}$$

c) (8 pts) The semantics of formulas is defined as follows:

$$\begin{array}{rcl} [\![T]\!] & = & true \\ [\![F]\!] & = & false \\ [\![f_1 \wedge f_2]\!] & = & [\![f_1]\!] \text{ andalso } [\![f_2]\!] \\ [\![f_1 \vee f_2]\!] & = & [\![f_1]\!] \text{ orelse } [\![f_2]\!] \end{array}$$

where andalso returns $true$ only if both arguments are $true$ (otherwise, $false$), and orelse returns $false$ only if both arguments are $false$ (otherwise, $true$).

Let $n(f)$ denote the number of occurrences of F in formula f. For example, $n(T \wedge (F \vee F)) = 2$. Complete the proof of the following property over every f:

$$n(f) = 0 \implies [\![f]\!] = true \qquad (1)$$

- (Base case 1) The first base case is when $f = T$. Because $n(f) = 0$ and $[\![f]\!] = true$, the property holds.
- (Base case 2) The other base case is when $f = F$. Because $n(f) = 1$, the premise of (1) is false. Therefore, the entire property (1) is true.
- (Inductive case 1) The first inductive case is when $f = f_1 \wedge f_2$. The inductive hypotheses (I.Hs) are

$$n(f_1) = 0 \implies [\![f_1]\!] = true \qquad (2)$$
$$n(f_2) = 0 \implies [\![f_2]\!] = true \qquad (3)$$

  If $n(f) = 0$, then $n(f_1) = \boxed{0}$ and $n(f_2) = \boxed{0}$ because there is no occurrence of F in f. By I.Hs (2) and (3), $[\![f_1]\!] = \boxed{\text{true}}$ and $[\![f_2]\!] = \boxed{\text{true}}$. By the definition of andalso, $[\![f]\!] = true$, which completes the proof for the case.

- (Inductive case 2) The other inductive case is when $f = f_1 \vee f_2$. The inductive hypothesis (I.H) is the above equations (2) and (3). If $n(f) = 0$, then $n(f_1) = \boxed{0}$ and $n(f_2) = \boxed{0}$ because there is no occurrence of F in f. By I.Hs (2) and (3), $[\![f_1]\!] = \boxed{\text{true}}$ and $[\![f_2]\!] = \boxed{\text{true}}$. By the definition of orelse, $[\![f]\!] = true$, which completes the proof for the case.

Problem 2 [Functional Programming] (20 pts)

a) (10 pts) Consider the following similar two functions written in OCaml.

```
let rec double_all l =
match l with
| [] -> []
| hd::tl -> (hd + hd) :: (double_all tl)

let rec dec_all l =
match l with
| [] -> []
| hd::tl -> (hd - 1) :: (dec_all tl)
```

With the following higher-order function `map`,

```
let rec map f l =
match l with
| [] -> []
| hd::tl -> (f hd)::(map f tl)
```

rewrite the two functions:

`let double_all l =` $\boxed{\text{map (fun x-> x + x) l}}$

`let dec_all l =` $\boxed{\text{map (fun x -> x - 1) l}}$

b) (10 pts) Let us define the type of natural numbers as follows:

```
type nat = Zero | Succ of nat
```

Complete the following definition of `int2nat : int -> nat`, which converts integers to natural numbers. For example, int2nat 3 evaluates to (Succ (Succ (Succ Zero))).

`let rec int2nat n =`
`if n = 0 then` $\boxed{\text{Zero}}$
`else` $\boxed{\text{Succ (int2nat (n-1))}}$

Problem 3 [Evaluation Rules] (20 pts)

Consider the following language.

$$E \to n \mid x \mid E+E \mid E-E \mid E! \mid \text{fib}(E)$$

where $E!$ denotes the factorial of E, and $\text{fib}(E)$ denotes the E-th Fibonacci number.

The factorial of 0 is 1 and the factorial of n for $n \geq 1$ is $n \times (n-1)!$. The Fibonacci numbers are inductively defined as follows:

$$\text{fib}(n) = \begin{cases} 1 & (n = 0 \text{ or } 1) \\ \text{fib}(n-1) + \text{fib}(n-2) & (n \geq 2) \end{cases}$$

Complete the following evaluation rules.

$$\frac{}{\rho \vdash n \Rightarrow n} \qquad \frac{}{\rho \vdash x \Rightarrow \rho(x)}$$

$$\frac{\rho \vdash E_1 \Rightarrow n_1 \quad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E_1 \Rightarrow n_1 \quad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 - E_2 \Rightarrow n_1 - n_2}$$

$$\frac{\rho \vdash E \Rightarrow n}{\rho \vdash E! \Rightarrow 1}\ n = 0$$

$$\frac{\rho \vdash E \Rightarrow n \quad \boxed{\rho \vdash (E-1)! \Rightarrow m}}{\rho \vdash E! \Rightarrow \boxed{\text{n} \times m}}\ n \geq 1$$

$$\frac{\rho \vdash E \Rightarrow n}{\rho \vdash \text{fib}(E) \Rightarrow 1}\ n = 0 \text{ or } 1$$

$$\frac{\rho \vdash E \Rightarrow n \quad \boxed{\rho \vdash \text{fib}(n-1) \Rightarrow n_1 \quad \rho \vdash \text{fib}(n-2) \Rightarrow n_2}}{\rho \vdash \text{fib}(E) \Rightarrow \boxed{\text{n}_1 + n_2}}\ n \geq 2$$

Problem 4 [Nameless Representation] (10 pts)

Write the nameless representation of the following program P:

```
let x = 2 in
  proc(y)(let z = x + y in proc(w)(w + z))
```

$\boxed{\textit{let } 2 \textit{ in proc } (\textit{let } \#1 + \#0 \textit{ in proc } \#0 + \#1)}$

which is $\mathsf{trans}(P)([])$. The trans function is defined as follows:

$$\begin{array}{rcl} \mathsf{trans}(n)(\rho) & = & n \\ \mathsf{trans}(x)(\rho) & = & \#n \quad (n \text{ is the first position of } x \text{ in } \rho) \\ \mathsf{trans}(E_1 + E_2)(\rho) & = & \mathsf{trans}(E_1)(\rho) + \mathsf{trans}(E_2)(\rho) \\ \mathsf{trans}(\mathtt{let}\ x = E_1\ \mathtt{in}\ E_2)(\rho) & = & \mathtt{let}\ \mathsf{trans}(E_1)(\rho)\ \mathtt{in} \\ & & \mathsf{trans}(E_2)(x :: \rho) \\ \mathsf{trans}(\mathtt{proc}(x)\ E)(\rho) & = & \mathtt{proc}\ \mathsf{trans}(E)(x :: \rho) \end{array}$$

Problem 5 [Scoping] (10 pts)

Assuming static scoping for procedures, consider the following two programs.

```
let f = let cnt = ref 0
        in proc (x) (cnt := !cnt + 1; !cnt)
in let a = (f 0)
   in let b = (f 0)
      in (a - b)
```

```
let f = proc(x) (let cnt = ref 0
                 in (cnt := !cnt + 1; !cnt))
in let a = (f 0)
    in let b = (f 0)
      in (a - b)
```

a) Write the evaluation result of the first program. $\boxed{-1}$

b) Write the evaluation result of the second program. $\boxed{0}$

Problem 6 [Evaluation strategy] (10 pts).

Consider lambda calculus with the normal order strategy and the following expression.

$$(\lambda x.\ (x\ x))\ (\lambda x.\ (x\ x))$$

a) Write the result after *one step* of β-reduction.

$\boxed{(\lambda x.\ (x\ x))\ (\lambda x.\ (x\ x))}$

b) Will the program eventually terminate? (write yes/no)

$\boxed{\textit{no}}$

Problem 7 [Type inference] (20 pts).

Consider the following program.

$$\underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{(\texttt{proc}\ (\underbrace{y}_{t_y})\ \underbrace{(\texttt{if}\ x\ \texttt{then}\ y\ \texttt{else}\ 1)}_{t_2})}_{t_1}}_{t_0}$$

a) Generate type equations.

```
t0 = tx -> t1
t1 = ty -> t2
tx = bool
ty = t2
t2 = int
```

b) Solve the equations using the unification algorithm, and write the final substitution.

```
t0 = bool -> int -> int
t1 = int -> int
tx = bool
ty = int
t2 = int
```

Problem 8 [Garbage Collection] (10 pts).

Recall the language with records, pointers, and automatic garbage collection we learned in class. Consider the following environment ρ and memory σ before GC:

$$\rho = \begin{bmatrix} x \mapsto l_1 \\ y \mapsto l_2 \\ z \mapsto l_3 \end{bmatrix} \qquad \sigma = \begin{bmatrix} l_1 \mapsto 0 \\ l_2 \mapsto \{a \mapsto l_3, b \mapsto l_1, c \mapsto l_5\} \\ l_3 \mapsto l_5 \\ l_4 \mapsto (x, E, [z \mapsto l_5]) \\ l_5 \mapsto 0 \\ l_6 \mapsto l_7 \\ l_7 \mapsto l_8 \\ l_8 \mapsto l_6 \end{bmatrix}$$

Describe the memory $\mathsf{GC}(\rho,\sigma)$ that can be obtained after GC:

$$\mathsf{GC}(\rho,\sigma) = \begin{bmatrix} l_1 \mapsto 0 \\ l_2 \mapsto \{a \mapsto l_3, b \mapsto l_1, c \mapsto l_5\} \\ l_3 \mapsto l_5 \\ l_5 \mapsto 0 \end{bmatrix}$$

Problem 9 [O/X questions] (20 pts).

Mark O for each correct statement (X for wrong statement).

a) The type checker for C programs is sound and complete. (X)

b) Manual memory management in C is difficult in general, leading to memory-leak, double-free, and use-after-free errors. (O)

c) Any lambda calculus expression can be translated into a Turing machine. (O)

d) A type system that always accepts input programs is sound. (X)

e) The type of `f` in the following OCaml code is `(int -> bool) -> int -> int -> int`. (O)

```
let rec sum_if_true test first second =
  (if test first then first else 0)
  + (if test second then second else 0)
```

f) The following function is tail-recursive. (O)

```
let rec f () = f ()
```

g) We cannot further remove syntactic sugars from Lambda calculus. (O)

h) Using eager evaluation, the following program terminates. (X)

```
letrec infinite(x) = (infinite x)
in let f = proc (x) (1)
   in  (f (infinite 0))
```

i) Determining the values of program variables is a dynamic property. (O)

j) Imperative languages encourage to use statements and loops, whereas functional languages encourage to use expressions and recursion. (O)