



Search Prioritization

Woosuk Lee

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Hanyang University

Limitations of Enumerative Search

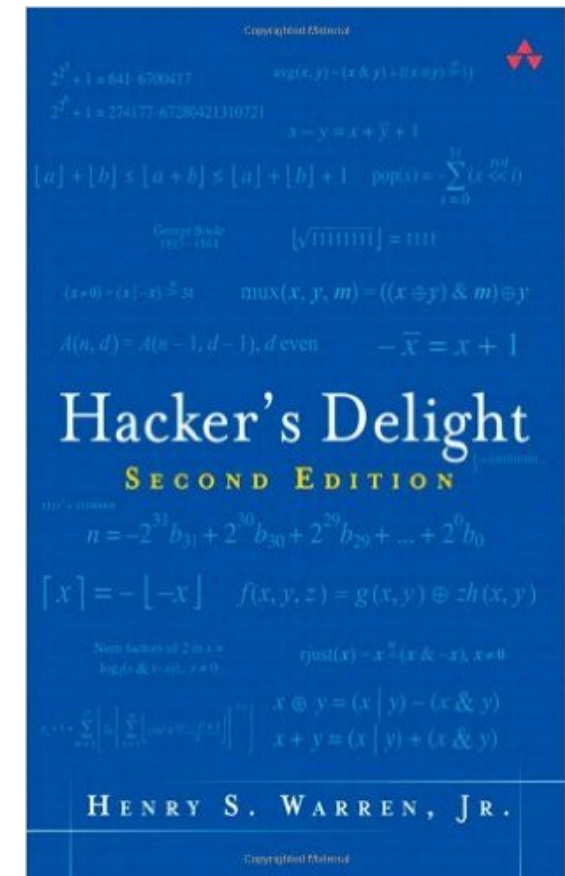
I. Limited Scalability

- Explore candidates in order of increasing size
 - Good for finding generalizable solutions (Occam's razor)
- What if desired solutions are large?
 - Search space exponentially grows
- Enumerative search wastes computation resources for exploring many “unlikely” candidates.

Example: Hacker's Delight

- Find a program transforming rightmost contiguous 1's into 0's (without a loop)
- Target : $f(x: \text{BitVec}) : \text{BitVec}$
- Syntactic constr.:

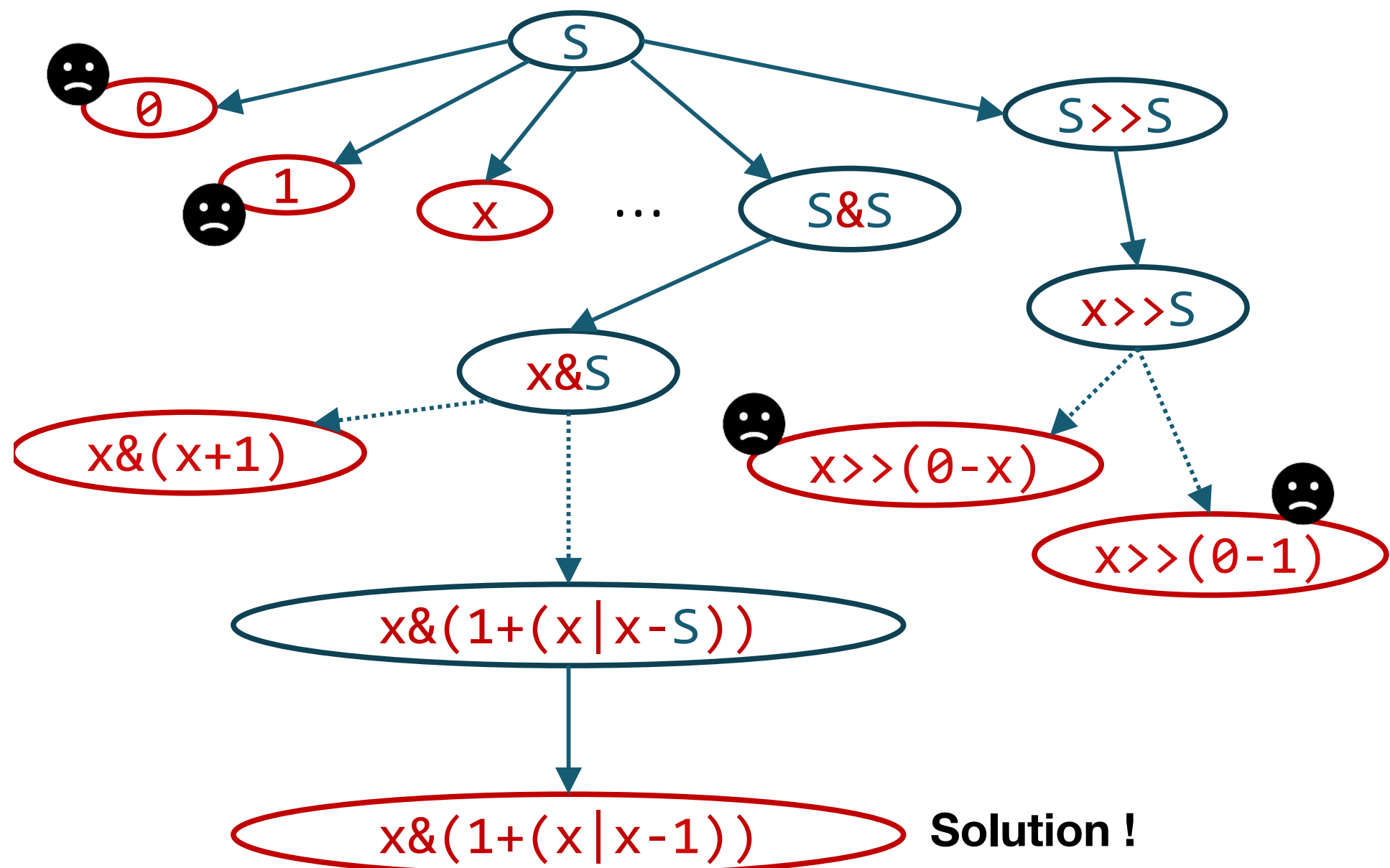
$S \rightarrow \emptyset \mid 1 \mid x \mid$
 $S + S$
 $S - S$
 $S \& S$
 $S \mid S$
 $S \ll S$
 $S \gg S$



- Semantic constr: $f(00101) = 00100, f(10110) = 10000 \dots$

Example: Hacker's Delight

Many “unlikely” candidates are explored by top-down search!



Limitations of Enumerative Search

2. Overfitting

- Despite Occam's razor, enumerative search does not guarantee most “likely” solutions.
- “likely”: generalizable beyond given I/O examples

Statistical Regularities in Programs

- Programs contain repetitive and predictable patterns [Hindle et al. ICSE'12]

```
for (i = 0; i < 100; ??)
```

- Statistical program models define a probability distribution over programs

$$Pr(?? \rightarrow i++ \mid \text{for } (i = 0; i < 100; ??)) = 0.80$$
$$Pr(?? \rightarrow i-- \mid \text{for } (i = 0; i < 100; ??)) = 0.01$$

– e.g., n-gram, neural network (e.g., LSTM), Sequence-based

probabilistic context-free grammar (PCFG),

probabilistic higher-order grammar (PHOG)...

Grammar-based

- Many applications: code completion, deobfuscation, program repair, etc.

Applications of Statistical Program Models

Input: code snippet
with holes

```
SmsManager smsMgr = SmsManager.getDefault();
int length = message.length();
if (length > MAX_SMS_MESSAGE_LENGTH) {
    ArrayList<String> msgList =
        smsMgr.divideMsg(message);
    ? {smsMgr, msgList} // (H1)
} else {
    ? {smsMgr, message} // (H2)
}
```



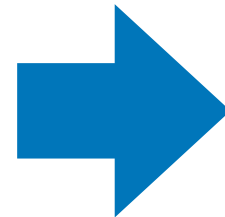
Output: holes completed with
(sequences) of method calls

```
SmsManager smsMgr = SmsManager.getDefault();
int length = message.length();
if (length > MAX_SMS_MESSAGE_LENGTH) {
    ArrayList<String> msgList =
        smsMgr.divideMsg(message);
    smsMgr.sendMultipartTextMessage(...msgList...);
} else {
    smsMgr.sendMessage(...message...);
}
```


Applications of Statistical Program Models

- Fixing syntactic errors

```
def evaluatePoly(poly, x):  
    a = 0  
    f = 0.0  
    for a in range(0, len(poly) - 1):  
        f = poly[a]*x**a+f  
        a += 1  
    return f
```



```
def evaluatePoly(poly, x):  
    a = 0  
    f = 0.0  
    while a < len(poly):  
        f = poly[a]*x**a+f  
        a += 1  
    return f
```

- Program = sequence of tokens
- Fix syntax errors using a skip-gram model

Exploiting Statistical Regularities

- Can we leverage statistical program models to accelerate program synthesis?
- Key Challenges
 - **Guided search:** How to guide the search given a statistical model?
 - **Learning models:** How to learn a good statistical model?

Euphony for Guiding Top-Down Enumerative Search

- Woosuk Lee, Kihong Heo, Rajeev Alur, Mayur Naik, Accelerating Search-Based Synthesis Using Learned Probabilistic Models, PLDI'18
- **Guided search:** A general approach to accelerate *CEGIS*-based program synthesis
 - by using a probabilistic model to guide the search towards likely programs
 - supports a wide range of models (e.g., n -gram, PCFG, PHOG, neural nets, ...)
- **Learning models:** Transfer learning-based method to mitigate overfitting
- <https://github.com/wslee/euphony>

Example SyGuS Problem

- Goal: a function f that replaces a hyphen (-) by a dot (.) in a given string x



Specification

Syntactic specification:

$$S \rightarrow x \mid \text{"-"} \mid \text{"."} \mid S + S \mid \text{Rep}(S, S, S)$$

String concatenation

$\text{Rep}(s, t1, t2)$: $t1$ in s is replaced by $t2$

Semantic specification:

$$f(\text{"-."}) = \text{".."} \wedge f(\text{"308-916"}) = \text{"308.916"} \wedge f(\text{"1"}) = \text{"1"}$$

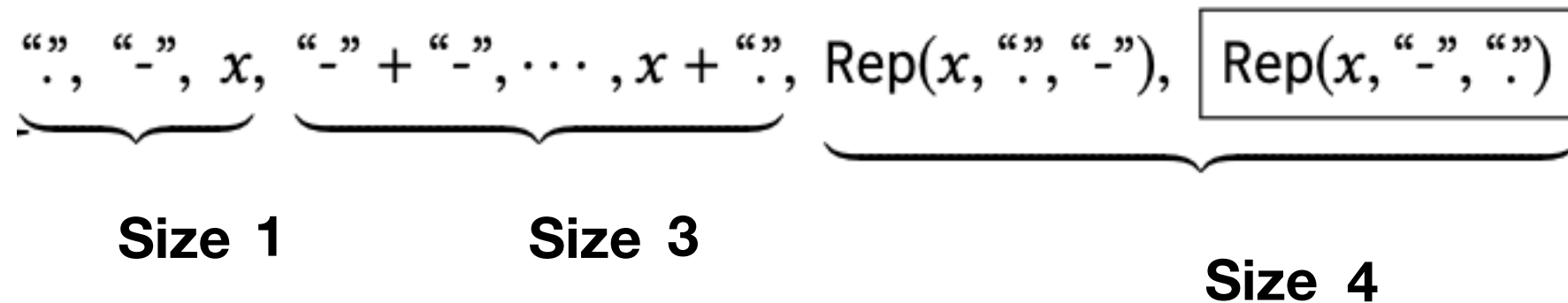


Solution

$$\text{Rep}(x, \text{"-"}, \text{"."})$$

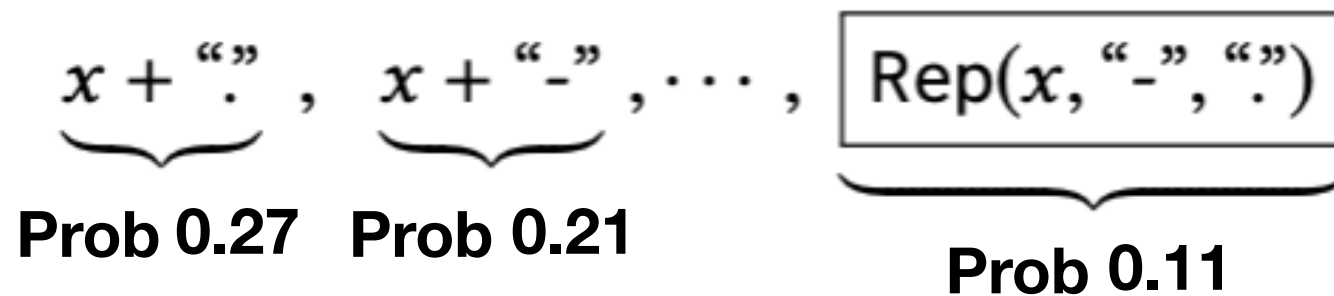
Guided Search

- Existing unguided enumerative search



- Unlikely candidates (e.g., $\text{ "-"} + \text{ "-"}$) are explored.

- Guided enumerative search



- Likely candidates are explored first, while preserving the existing pruning optimizations

Guided Search

- **Model learning**
- Guided enumerative search

(Grammar-based) Statistical Program Models

- Given CFG $\langle N, \Sigma, R, S \rangle$
- and an incomplete program: $(N \cup \Sigma)^*$ (i.e., *sentential form*),
- provides a probability for each production rule applicable next (usually on the leftmost nonterminal)
 - $Pr(\textit{production rule} \mid \textit{sentential form})$

(Grammar-based) Statistical Program Models

- Determines a probability of a given program
- E.g., probability of $x + \text{"."}$

$$\begin{array}{ccccccc} \underline{S} & \Longrightarrow & \underline{S + S} & \Longrightarrow & \underline{x + S} & \Longrightarrow & \underline{x + \text{"."}} \\ \vdots & & \vdots & & \vdots & & \vdots \\ Pr(S \rightarrow S + S \mid S) & & \times Pr(S \rightarrow x \mid S + S) & & \times Pr(S \rightarrow \text{"."} \mid x + S) & & \end{array}$$

Example

- *Probabilistic context-free grammar (PCFG)*

	$A \rightarrow \beta$	P
S	\rightarrow “.”	0.2
S	\rightarrow “_”	0.2
S	\rightarrow x	0.1
S	\rightarrow $S + S$	0.1
S	\rightarrow $\text{Rep}(S, S, S)$	0.4

- Limitation: context around the place where a rule is applied is not considered \rightarrow imprecise

A Uniform Interface to Statistical Program Models

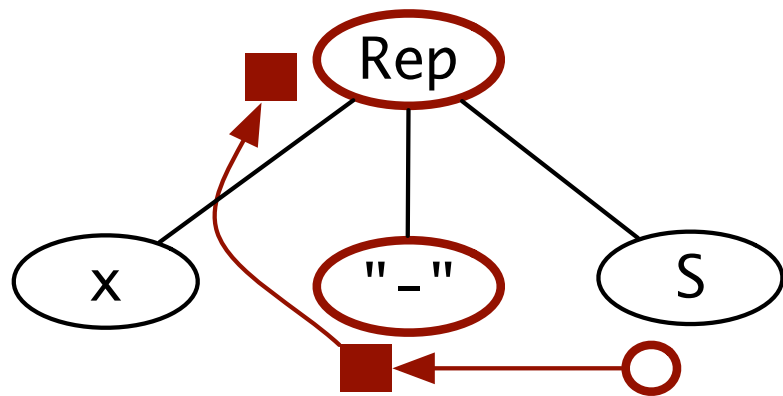
- A statistical program model is $G_q = \langle G, C, p, q \rangle$ for a given CFG $G = \langle N, \Sigma, R, S \rangle$
- C — set of *contexts*
- $p : (N \cup \Sigma)^* \rightarrow C$ — Given a sentential form, for extracting contextual information around the next hole (i.e., nonterminal) to be filled
- $q : R \times C \rightarrow \mathbb{R}^+$ — Considering contextual information, for determining a probability for production rule

Contexts

- Sequence of terminal/nonterminal symbols
- E.g., 2-gram

$$p(x+S) = [+ , x]$$

- E.g., Sibling and parent nodes



$$p(\text{Rep}(x, \text{"-"}, S)) = [\text{"-"}, \text{Rep}]$$

Example

Probabilistic Higher-order Grammar (PHOG) — the model used by Euphony

$$A[\textit{context}] \rightarrow \beta$$

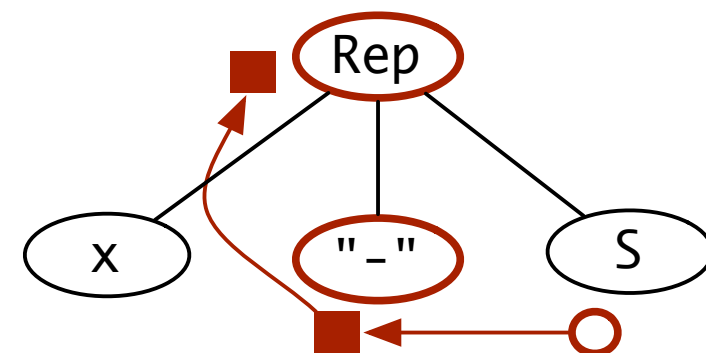
P

$S["-", \text{Rep}]$	\rightarrow	$“.”$	0.72
$S["-", \text{Rep}]$	\rightarrow	$“-”$	0.001
$S["-", \text{Rep}]$	\rightarrow	x	0.12
$S["-", \text{Rep}]$	\rightarrow	$S + S$	0.02
$S["-", \text{Rep}]$	\rightarrow	$\text{Rep}(S, S, S)$	0.139

...

PHOG when *context* is symbols at
left sibling and **parent**

$$\Pr(S \rightarrow “.” \mid \text{Rep}(“x”, “-”, S)) = 0.72$$



Learning a PHOG

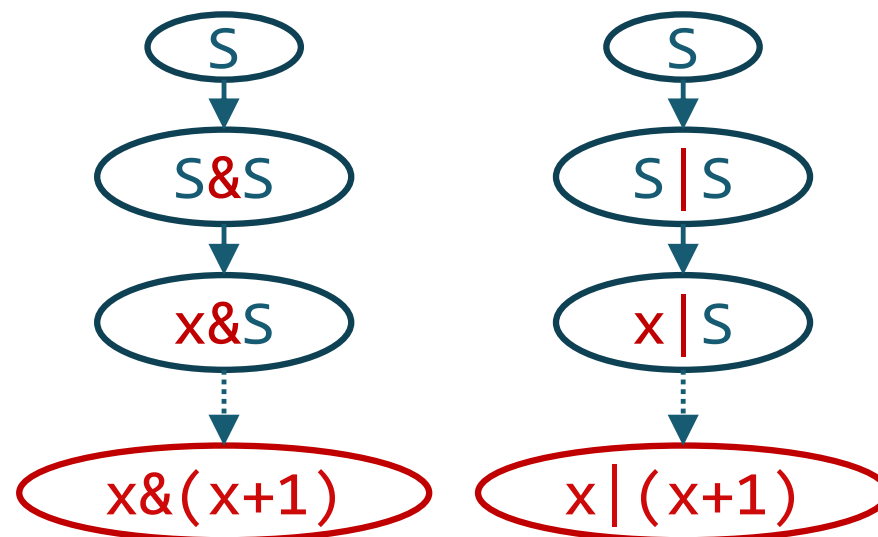
CFG +

Corpus

$x \& (x+1)$
 $x | (x-1)$
 x
 $x \& (x+x)$
 $x \& (1+(x | x-1))$
 \dots



ASTs / Paths



- From *derivation sequences* of training programs, count occurrences of each rule application under certain contexts
 - E.g., From derivation sequence $x \& S \implies x \& I$, $x | S \implies x | I$, production rule $S \rightarrow I$ is counted twice when sibling is x
- Prob. of $\alpha \rightarrow \beta$ under context γ : $q(\alpha[\gamma] \rightarrow \beta) = \frac{\text{Count}(\alpha[\gamma] \rightarrow \beta)}{\text{Count}(\alpha[\gamma])}$

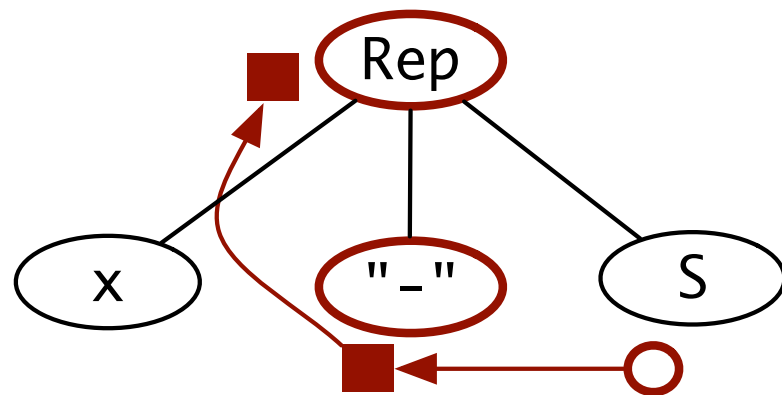
Learning a PHOG

- How to determine which contexts matter?
- The p function: program written in the TCond language

TCond $\rightarrow \epsilon \mid \text{Write TCond} \mid \text{MoveOp TCond}$

MoveOp $\rightarrow \text{Up} \mid \text{Left} \mid \text{Right} \mid \text{DownFirst} \mid \text{DownLast}$

- E.g., collect contents of sibling and parent nodes



Left · Write · Up · Write

Learning a PHOG

- Given training programs \mathcal{D} , Find TCond program s.t.

$$p_{best} = \arg \min_{p' \in \text{TCOND}} \text{cost}(\mathcal{D}, p').$$

sum of negative log probabilities of training programs using p' for extracting contexts

- By using a genetic algorithm

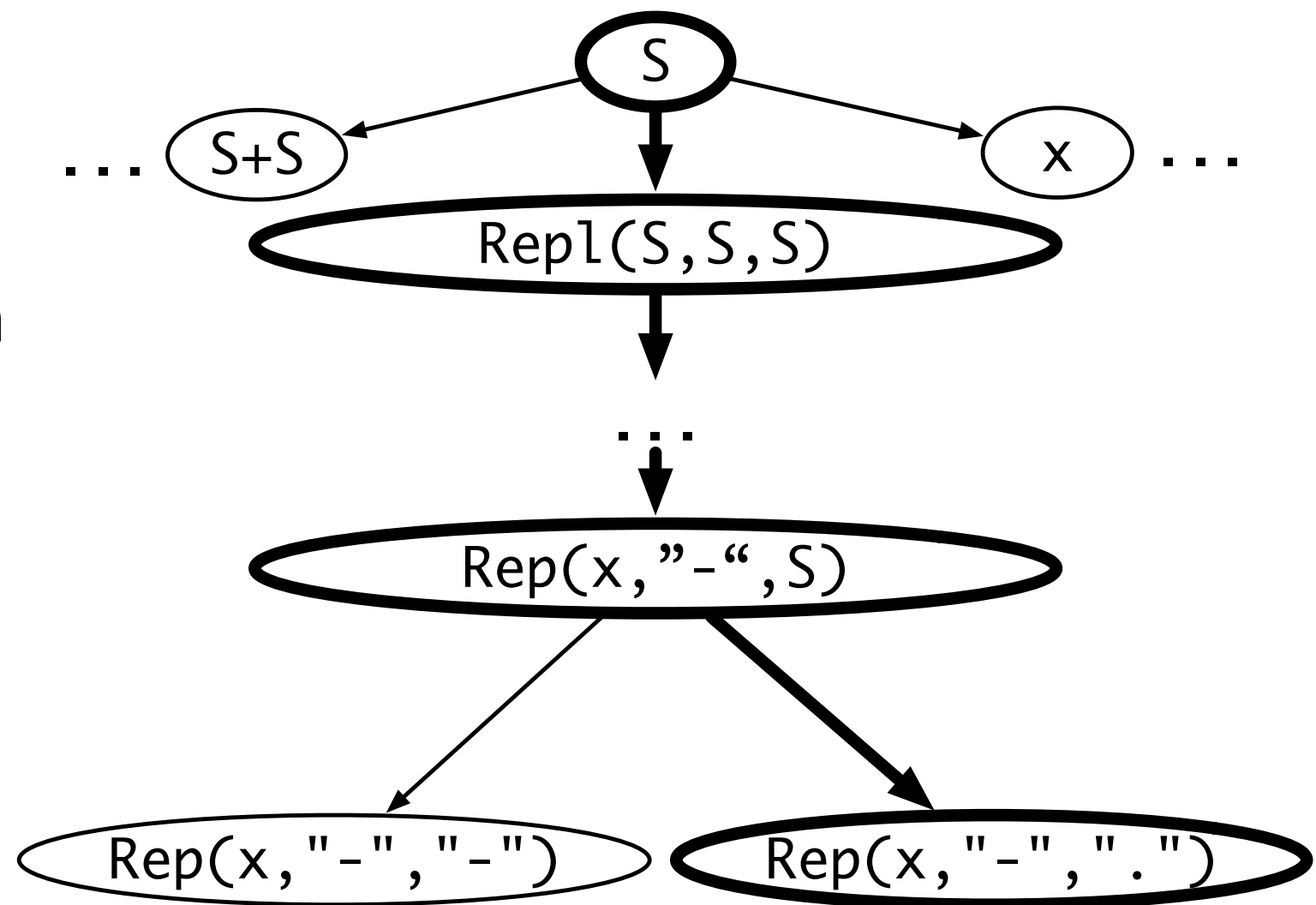
Guided Search

- Model Learning
- **Guided enumerative search**

Guided Search as Path Finding

Construct a directed weighted graph from a given CFG

- Nodes: sentential forms
- Edges: derivations between sentential forms
- Terminal nodes: sentences

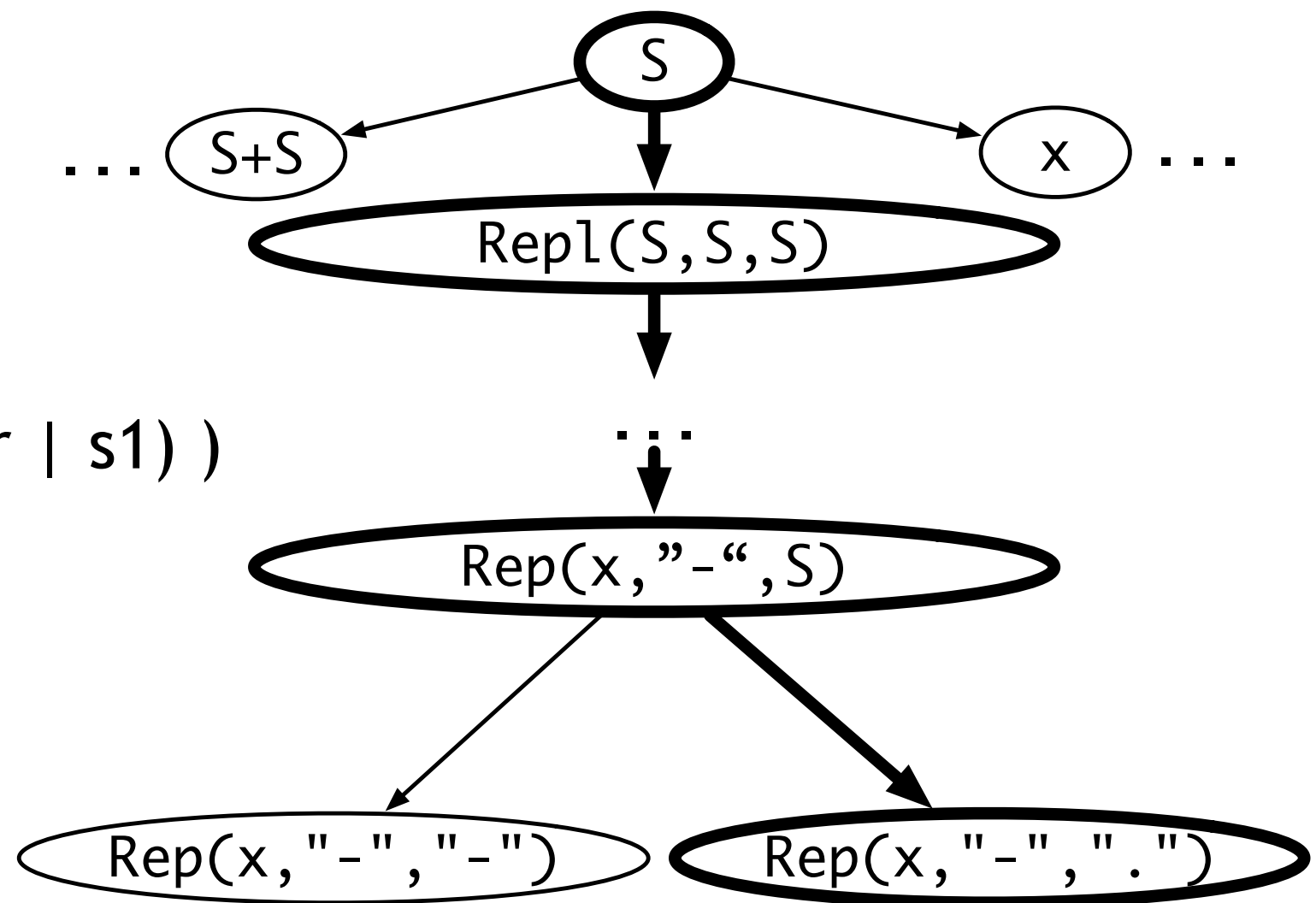


Guided Search as Path Finding

Construct a directed weighted graph from a given CFG

- Weights:

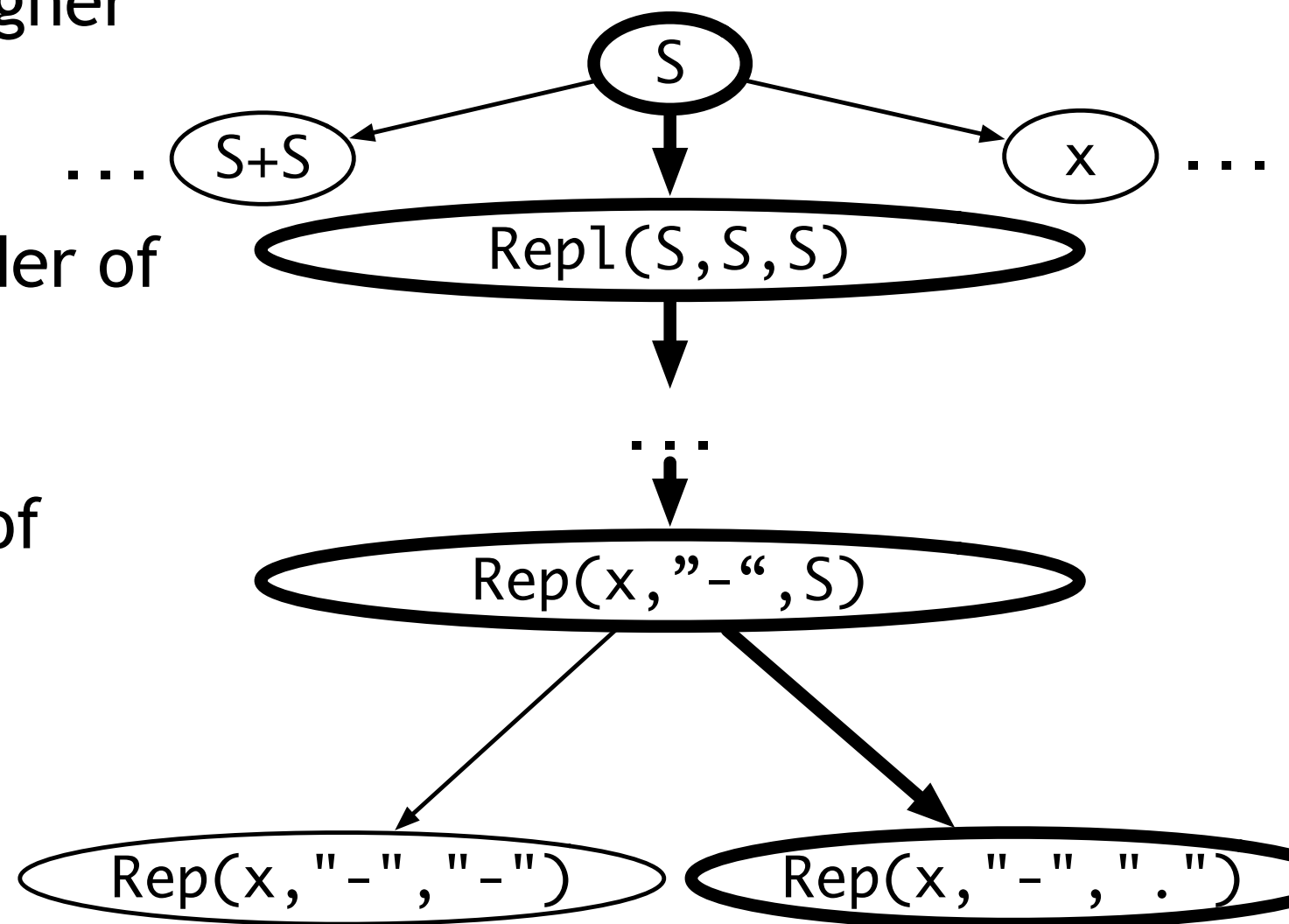
- $w(s1 \rightarrow s2) = -\log (Pr (r | s1))$



Guided Search as Path Finding

Construct a directed weighted graph from a given CFG

- Goal: explore candidates of higher probabilities first
- = enumerating programs in order of decreasing probability
- = enumerating paths in order of increasing distance



Unguided Top-Down Search

TopDown (grammar $G = \langle N, \Sigma, R, S \rangle$, spec Φ) :

$Q := \{S\}$

while $Q \neq \emptyset$:

 remove p from Q

if $\Phi(p)$: **return** p

$P' := \text{Unroll}(G, p)$

forall $p' \in P'$:

$Q := Q.\text{Enqueue}(p')$

Unroll (grammar G , spec Φ) :

$P' := \emptyset$

forall $A \in p$:

forall $A \rightarrow B \in R$:

$p' := p[B/A]$

$P' := P' \cup \{p'\}$

return P'

Guided Top-Down Search

TopDown (grammar $G = \langle N, \Sigma, R, S \rangle$, spec Φ) :

$Q := \{(S, 0)\}$

Candidates are with their distances from the root (S)

while $Q \neq \emptyset$:

remove (p, d) whose d is minimal from Q

if $\Phi(p)$: **return** p

$P' := \text{Unroll}(G, p, d)$

Pick one closest to S

forall $p' \in P'$:

$Q := Q.\text{Enqueue}(p')$

Unroll (grammar G , program p , distance d) :

$P' := \emptyset$

forall $A \in p$:

forall $A \rightarrow B \in R$:

$p' := p[B/A]$

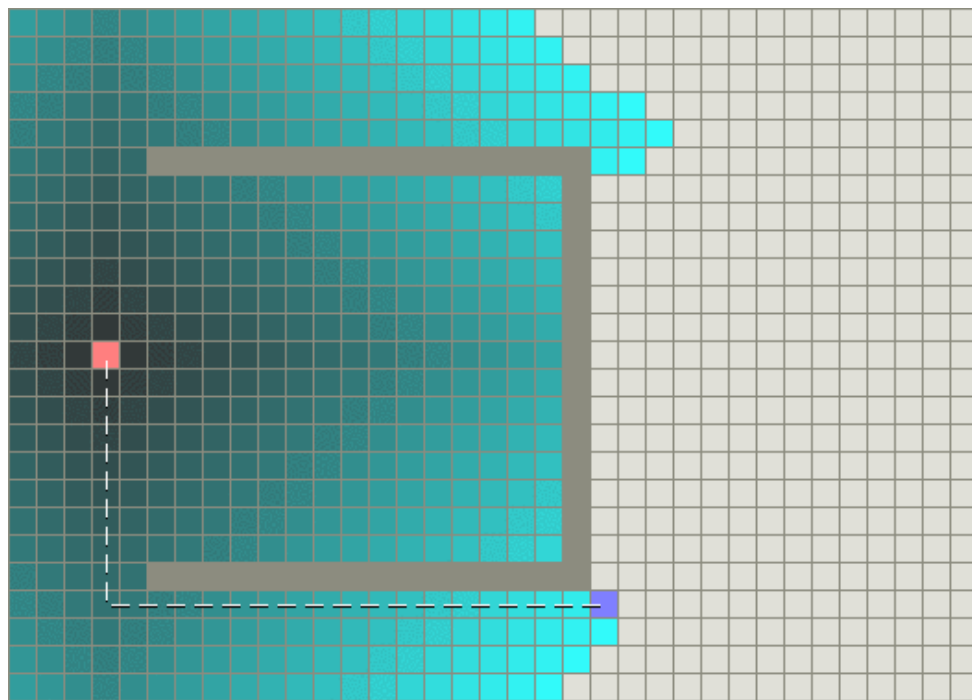
$P' := P' \cup \{(p', d + w(p, p'))\}$

Add new candidates to the queue with their distances from S

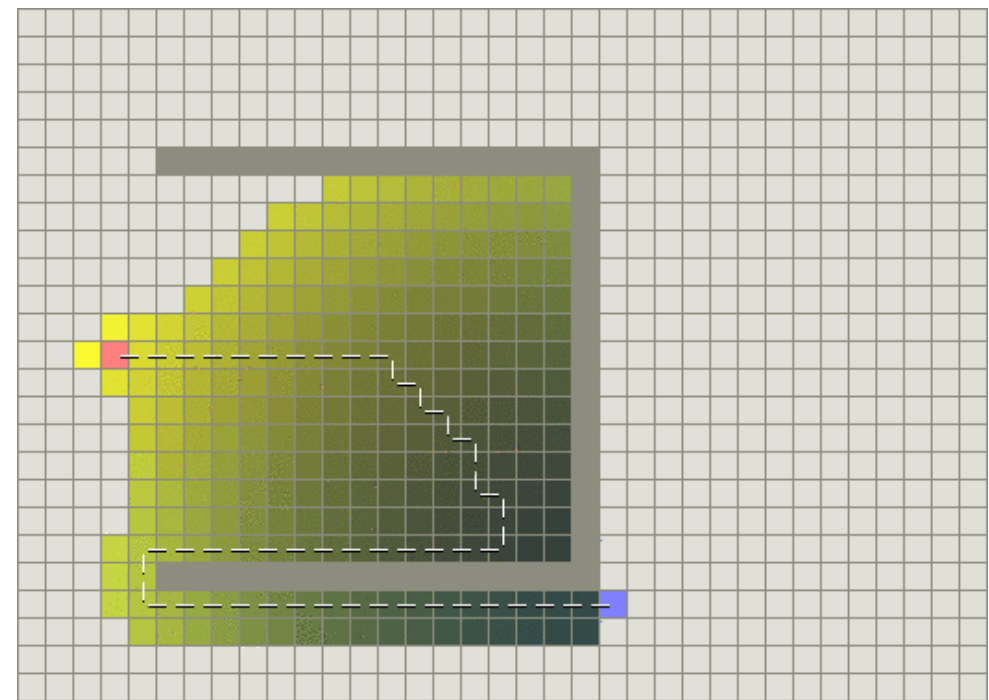
return P'

Better Guided Top-Down Search

- The previous algorithm is based on Dijkstra algorithm
- We can use A^* , which is better.



Dijkstra



A^*

Red: start node
Blue: goal node
GreenYellow: explored nodes

A* Search

- Dijkstra: picks one closest to the root
- A*: picks one of the *estimated* shortest path (= distance from the root + guessed future distance to the closest goal node)
- Often infeasible to compute the exact future distance — an *under-approximation* is used.
- Heuristic function $g : \text{Node} \rightarrow \text{Guessed Future distance}$
- A* finds the shortest paths if the heuristic function always underestimates future distances.

Guided Top-Down Search (improved)

TopDown (grammar $G = \langle N, \Sigma, R, S \rangle$, spec Φ):

$Q := \{ (S, 0, g(S)) \}$

Gussed future distance
 $g : (N \cup \Sigma)^* \rightarrow \mathbb{R}^+$

while $Q \neq \emptyset$:

remove (p, d, h) whose $d + h$ is minimal from Q

if $\Phi(p)$: **return** p

$P' := \text{Unroll}(G, p, d)$

forall $p' \in P'$:

$Q := Q.\text{Enqueue}(p')$

Pick one of the estimated shortest path
(distance so far + future distance)

Unroll (grammar G , program p , distance d):

$P' := \emptyset$

forall $A \in p$:

forall $A \rightarrow B \in R$:

$p' := p[B/A]$

$P' := P' \cup \{ (p', d + w(p, p'), g(p')) \}$

return P'

Add new candidates to the queue
with their gussed future distances

How to compute g ?

- $n \overset{r}{\rightsquigarrow} s$: path from n to s

- $w(n \overset{r}{\rightsquigarrow} s)$: distance of the path from n to s

- Ideal heuristic function:

$$g^*(n) = \min_{s \in \Sigma^*, n \overset{r}{\rightsquigarrow} s} w(n \overset{r}{\rightsquigarrow} s)$$

- which is infeasible (\because possibly infinitely many goal nodes reachable from n)

How to compute g ?

- Uses an underapproximation
- Compute the h function satisfying the following condition

$$\forall A \in N. h(A) = \max_{A \rightarrow \beta \in R, c \in C} \left(q(A \rightarrow \beta \mid c) \times \prod_{\beta_i \in N} h(\beta_i) \right)$$

1) start with $h(A) = 0$ for all A

2) repeatedly update h according to the above equation until saturation

- E.g., Consider the following PCFG

$$S \rightarrow aSb \quad (0.9)$$

$$S \rightarrow c \quad (0.1)$$

1st iteration: $h(S) = \max(0.9 \times 0, 0.1) = 0.1.$

The highest probability of program derivable from S is 0.1

2nd iteration: $h(S) = \max(0.9 \times 0.1, 0.1) = 0.1.$

How to compute g ?

- Define the following function using the h function

$$g(n) = \begin{cases} 0 & (n \in \Sigma^*) \\ -\sum_{n_i \in N} \log_2 h(n_i) & (\text{otherwise}) \end{cases}$$

n_i : i -th symbol in n

- This heuristic function is correct (why?):

$$\forall n \in (N \cup \Sigma)^*. g(n) \leq g^*(n).$$

Overfitting

- The guided search quickly finds the solution

$\text{Rep}(x, \text{"-"}, \text{"."})$.

- What if a similar problem of the following semantic constraint is given?

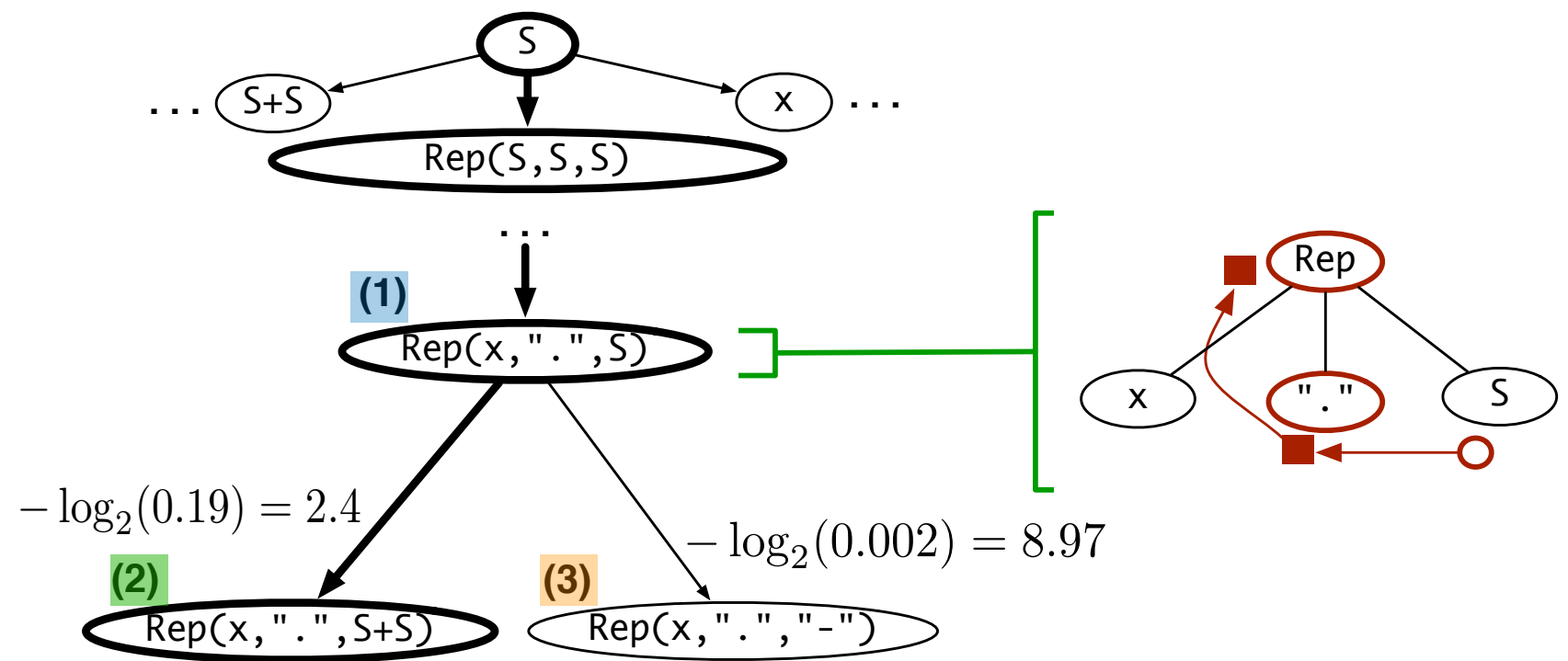
$$f(\text{"12.31"}) = \text{"12-31"} \wedge f(\text{"01.07"}) = \text{"01-07"}.$$

Solution: $\text{Rep}(x, \text{"."}, \text{"-"})$

Overfitting

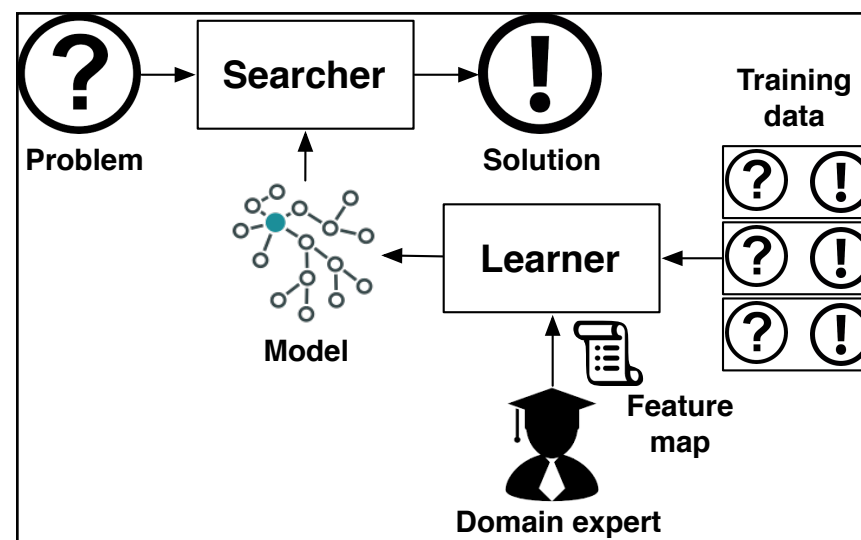
- Suppose $\text{Rep}(x, ".", S)$ (node **(1)**) is currently explored.
- Using the PHOG we have, node **(2)** is preferred above **(3)** as the next candidate

		$A[\text{context}] \rightarrow \beta$	P
$S["-", \text{Rep}]$	\rightarrow	"."	0.72
$S["-", \text{Rep}]$	\rightarrow	"_"	0.001
$S["-", \text{Rep}]$	\rightarrow	x	0.12
$S["-", \text{Rep}]$	\rightarrow	$S + S$	0.02
...			
		P	
$S[".", \text{Rep}]$	\rightarrow	"."	0.001
$S[".", \text{Rep}]$	\rightarrow	"_"	0.002
$S[".", \text{Rep}]$	\rightarrow	x	0.01
$S[".", \text{Rep}]$	\rightarrow	$S + S$	0.19
...			



- Because statistical models like PHOG only consider *syntactic information*.

Transfer Learning



- Training data: solutions of existing synthesis problems
- Testing data: solutions of unseen synthesis problems
- They may follow different probability distributions because of diverse semantic specifications.
- Transfer learning reduces discrepancy between the probability distributions of training and testing data

Transfer Learning

- Spec: $f("-.") = ".." \wedge f("308-916") = "308.916" \wedge f("1") = "1"$

Solution: $\text{Rep}(x, "-", ".")$

Constant string that appear in the inputs

Constant string that appear in the outputs

- Spec: $f("12.31") = "12-31" \wedge f("01.07") = "01-07"$.

Solution: $\text{Rep}(x, ".", "-")$

Constant string that appear in the inputs

Constant string that appear in the outputs

Transfer Learning

- Spec: $f("-.") = ".." \wedge f("308-916") = "308.916" \wedge f("1") = "1"$

$$\text{Rep}(x, "-", ".") \longrightarrow \text{Rep}(x, \text{const}_I, \text{const}_O)$$

Constant string that appear in the
inputs

Constant string that appear in the
outputs

- Spec: $f("12.31") = "12-31" \wedge f("01.07") = "01-07"$.

$$\text{Rep}(x, ".", "-") \longrightarrow \text{Rep}(x, \text{const}_I, \text{const}_O)$$

Now the solutions of the two problems become equal

Types of Constants

- I : Input examples O : Output examples
- $const_{IO}$: constants that appear in I and O
- $const_I$: constants that appear in I
- $const_O$: constants that appear in O
- $const_{\perp}$: constants that appear in neither I nor O

Pivot PHOG

$S \rightarrow x \mid S + S$
 $\quad \mid \text{Rep}(S, S, S)$
 $\quad \mid \text{const}_{IO} \mid \text{const}_I$
 $\quad \mid \text{const}_O \mid \text{const}_\perp$

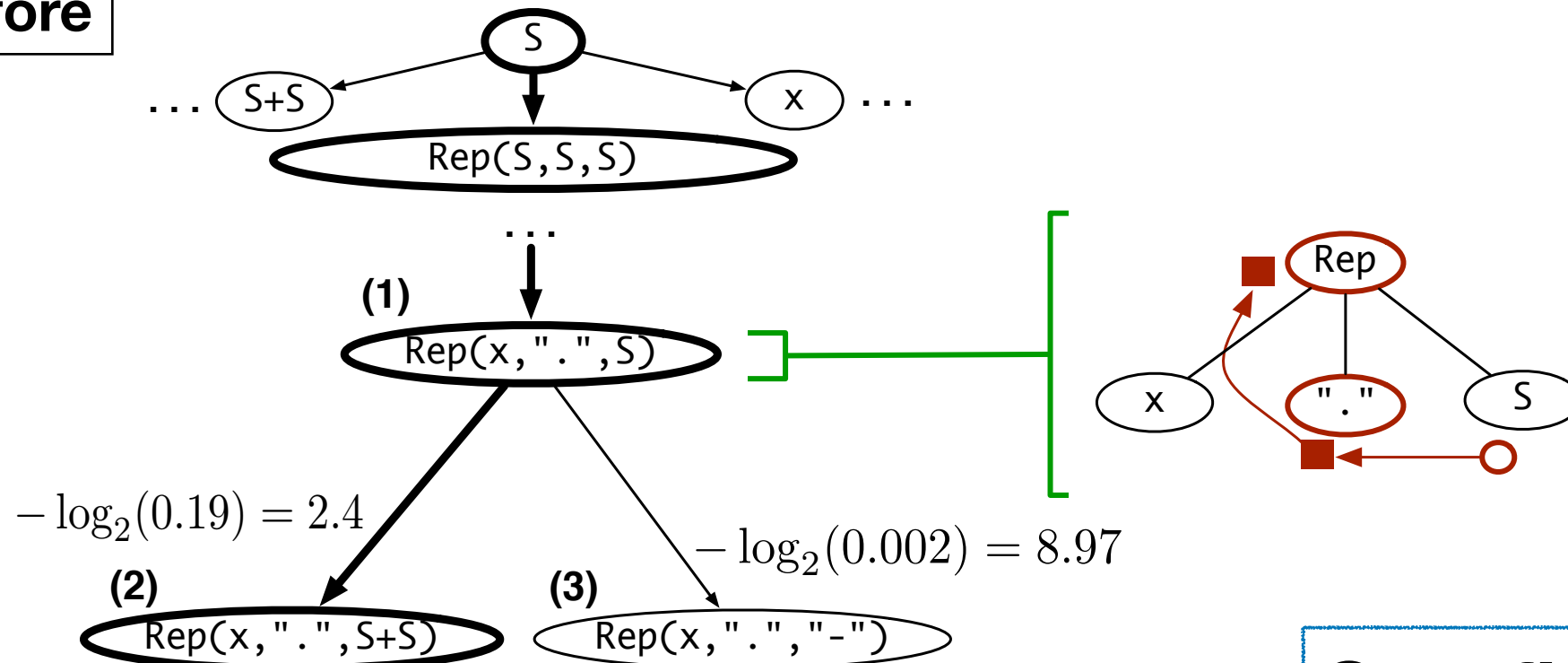
(a) A pivot grammar for string manipulation tasks

		$A[\textit{context}^\#] \rightarrow \beta^\#$	P
	$S[\text{const}_I, \text{Rep}]$	$\rightarrow \text{const}_O$	0.72
	$S[\text{const}_I, \text{Rep}]$	$\rightarrow \text{const}_I$	0.001
	$S[\text{const}_I, \text{Rep}]$	$\rightarrow x$	0.12
	$S[\text{const}_I, \text{Rep}]$	$\rightarrow S + S$	0.02
	...		P
	$S[\text{const}_O, \text{Rep}]$	$\rightarrow \text{const}_O$	0.001
	$S[\text{const}_O, \text{Rep}]$	$\rightarrow \text{const}_I$	0.002
	$S[\text{const}_O, \text{Rep}]$	$\rightarrow x$	0.01
	$S[\text{const}_O, \text{Rep}]$	$\rightarrow S + S$	0.19
	...		

(b) A pivot PHOG learned using the pivot grammar

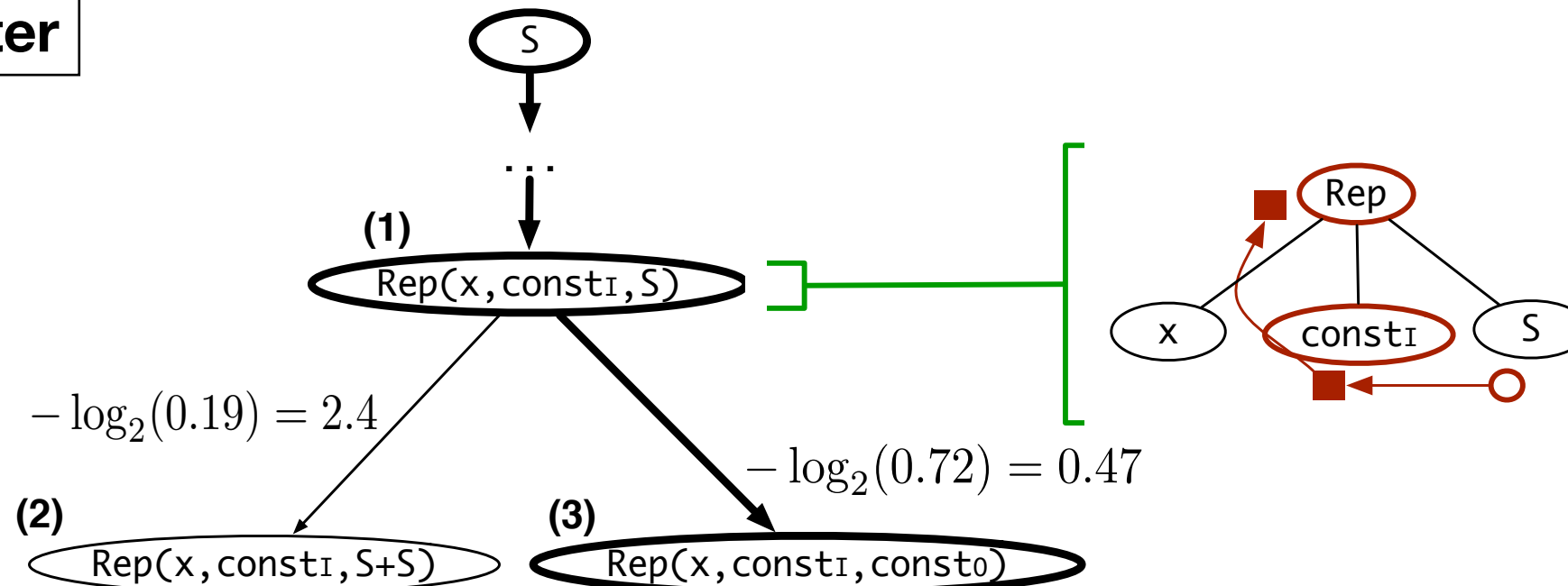
Guided Search with a Pivot PHOG

Before



Overfitting avoided!

After



Other Examples of Exploiting Spec

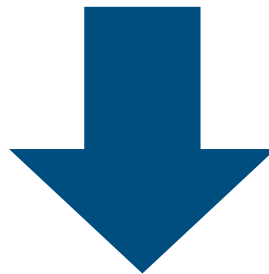
An input-output example:

Input:

`[-17, -3, 4, 11, 0, -5, -9, 13, 6, 6, -8, 11]`

Output:

`[-12, -20, -32, -36, -68]`



```
a ← [int]
b ← FILTER (<0) a
c ← MAP (*4) b
d ← SORT c
e ← REVERSE d
```

Other Examples of Exploiting Spec

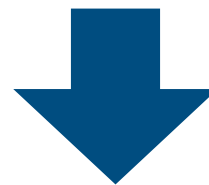
An input-output example:

Input:

`[-17, -3, 4, 11, 0, -5, -9, 13, 6, 6, -8, 11]`

Output:

`[-12, -20, -32, -36, -68]`



Neural net inference

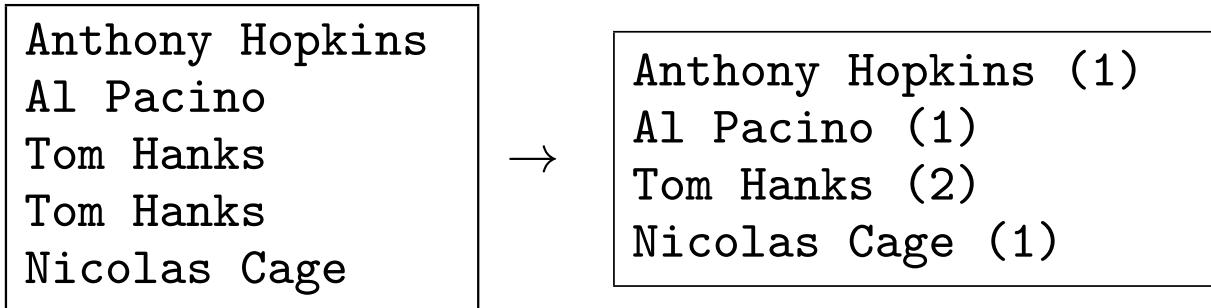
(+1)	(-1)	(*2)	(/2)	(*1)	(**2)	(*3)	(/3)	(*4)	(/4)	(>0)	(>0)	(%2==1)	(%2==0)	HEAD	LAST	MAP	FILTER	SORT	REVERSE	TAKE	DROP	ACCESS	ZIPWITH	SCANL1	+	.	*	MIN	MAX	COUNT	MINIMUM	MAXIMUM	SUM
.0	.0	.1	.0	.0	.0	.0	.0	1.0	.0	.0	1.0	.0	.2	.0	.0	1.0	1.0	1.0	.7	.0	.1	.0	.4	.0	.0	.1	.0	.2	.1	.0	.0	.0	.0



DFS + Sort and add + ...

```
a ← [int]
b ← FILTER (<0) a
c ← MAP (*4) b
d ← SORT c
e ← REVERSE d
```

Other Examples of Exploiting Spec



**Syntactic features
of I/O examples**

Feature	Answ
Duplicated lines in input but not output?	Y
Parentheses in output but not input?	Y
Numbers on each line in output but not input?	Y
...	

Solution

$$f(x) = \text{dedup}(\text{concatLists}(x, " ", \text{concatLists}("(" , \text{count}(x, x), ")"))).$$

Infer PCFG

**Enumeration in order of
decreasing probability**

Production	Probability	Production	Probability
$P \rightarrow \text{join}(\text{LIST}, \text{DELIM})$	1	$\text{CAT} \rightarrow \text{LIST}$	0.7
$\text{LIST} \rightarrow \text{split}(x, \text{DELIM})$	0.3	$\text{CAT} \rightarrow \text{DELIM}$	0.3
$\text{LIST} \rightarrow \text{concatList}(\text{CAT}, \text{CAT}, \text{CAT})$	0.1	$\text{DELIM} \rightarrow "\backslash n"$	0.5
$\text{LIST} \rightarrow \text{concatList}("(" , \text{CAT}, ")")$	0.2	$\text{DELIM} \rightarrow " "$	0.3
$\text{LIST} \rightarrow \text{dedup}(\text{LIST})$	0.2	$\text{DELIM} \rightarrow "("$	0.1
$\text{LIST} \rightarrow \text{count}(\text{LIST}, \text{LIST})$	0.2	$\text{DELIM} \rightarrow ")"$	0.1

Evaluation

- Benchmarks
 - **1,167** problems used in SyGuS annual competitions
- Baselines
 - **EUSolver** (general-purpose): winner of SyGuS competition
 - **FlashFill** (domain-specific): string processing in spreadsheets

Benchmarks

	A	B	C	D
1	Number	Phone		
2	02082012225	020-8201-2225		
3	02072221236	020-7222-1236		
4	0208123654	020-8123-654		
5	0207236523	020-7236-523		
6	02082012222	020-8201-2222		
7				
8				

STRING: End-user Programming
205 problems

complement

```
~ 010100011101011100000000000001111
   1010111000101000111111111110000
```

bitwise and

```
010100011101011100000000000001111
& 00110001011011100011000101101110
   00010001010001100000000000001110
```

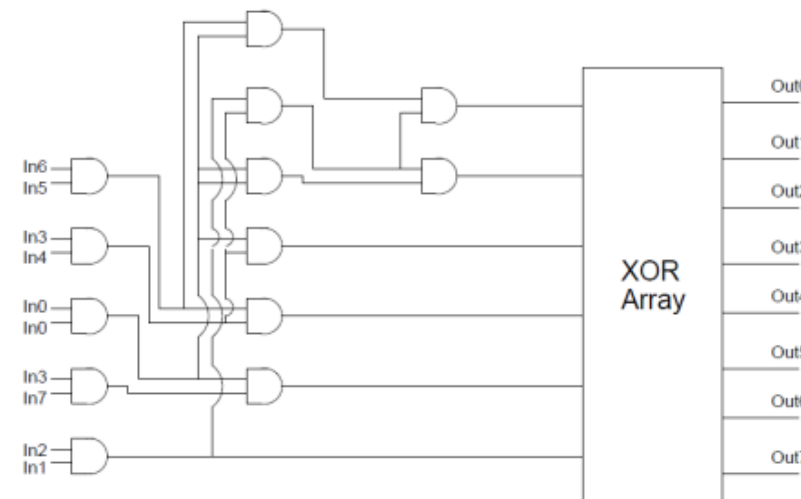
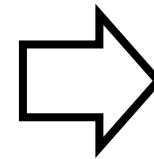
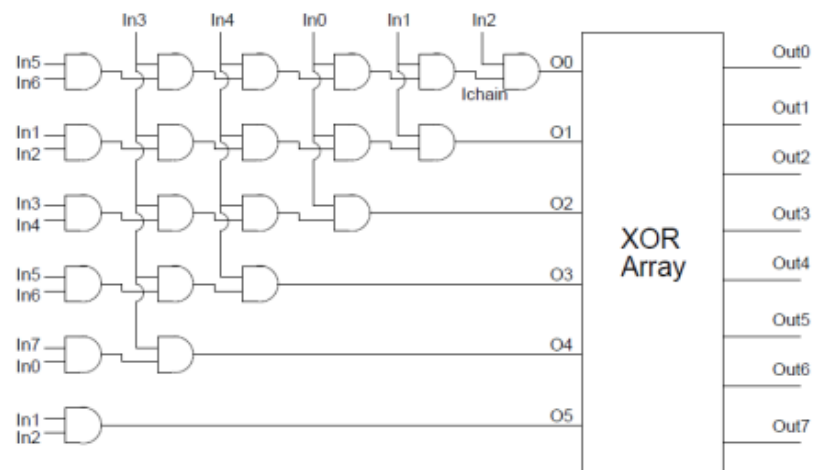
bitwise or

```
010100011101011100000000000001111
| 00110001011011100011000101101110
   011100011111111110011000101101111
```

bitwise xor

```
010100011101011100000000000001111
^ 00110001011011100011000101101110
   01100000101110010011000101100001
```

BITVEC: Efficient low-level algorithm
750 problems

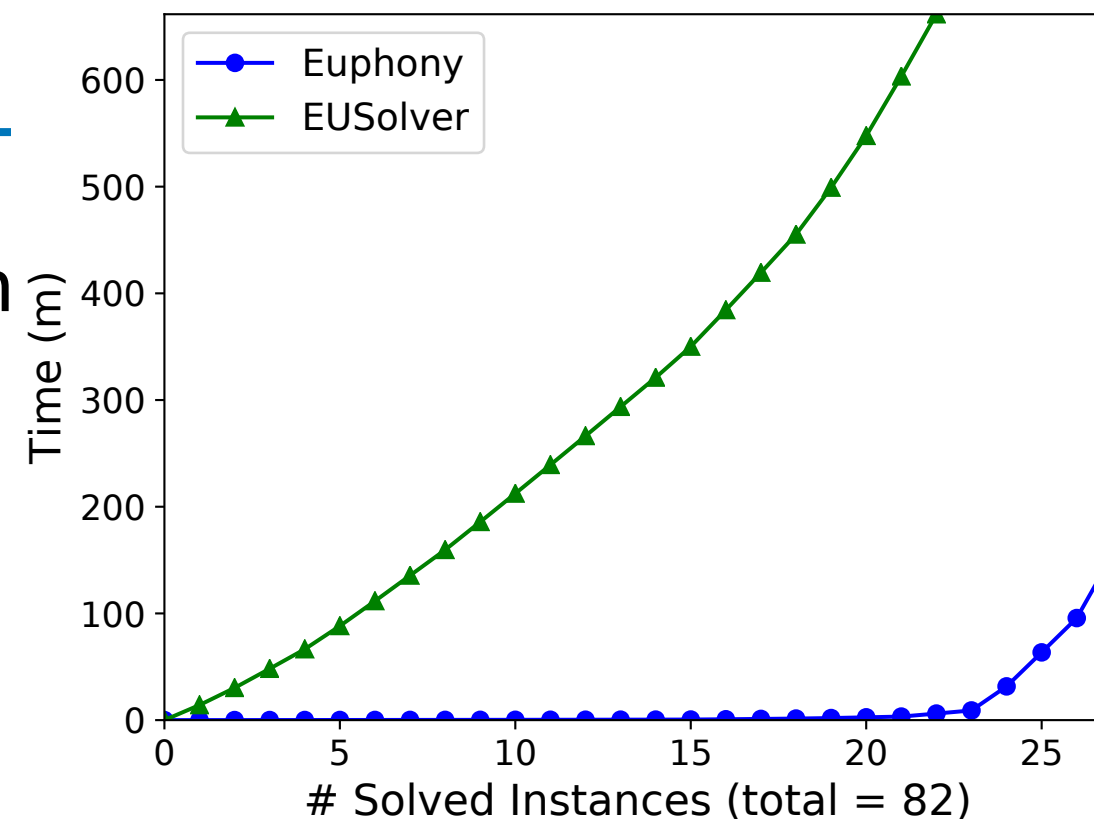


CIRCUIT: Attack-resistant crypto circuits
212 problems

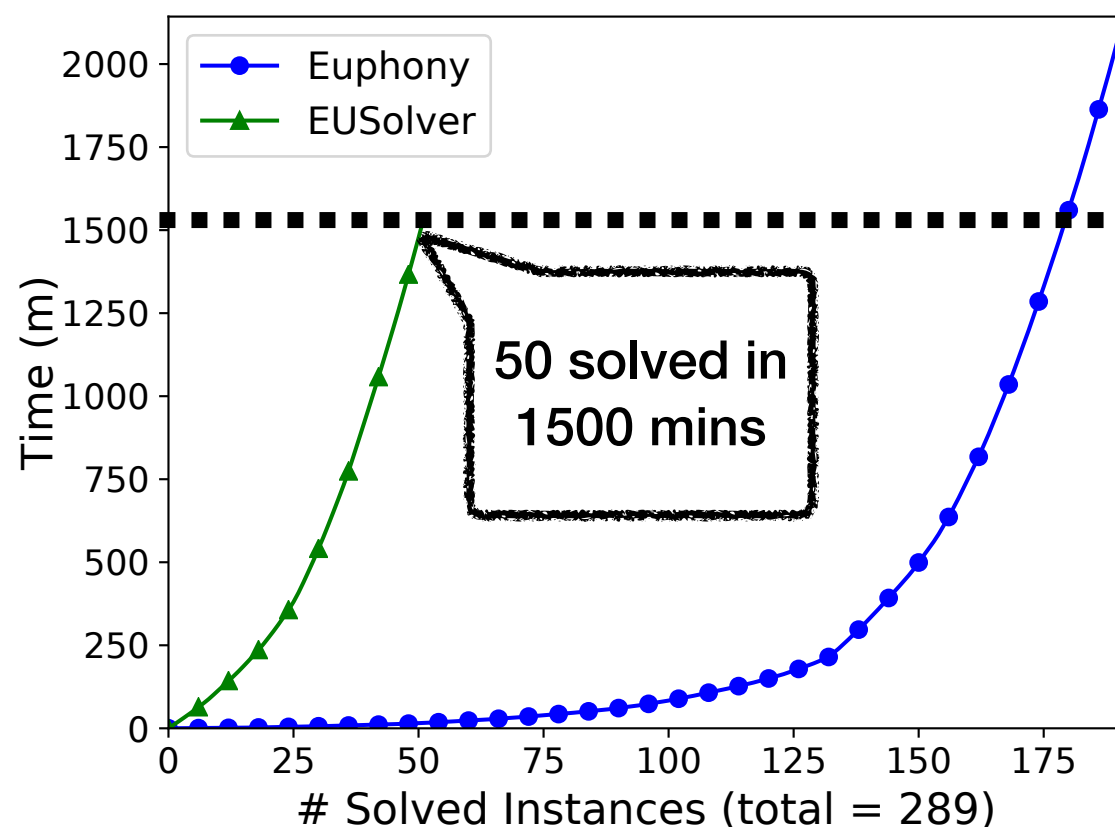
vs. EUSolver

- Training: **762** solved by EUSolver in 10 min
- Testing: **405** (timeout: 1 hour)
- # solved: Euphony **236**, EUSolver **87**

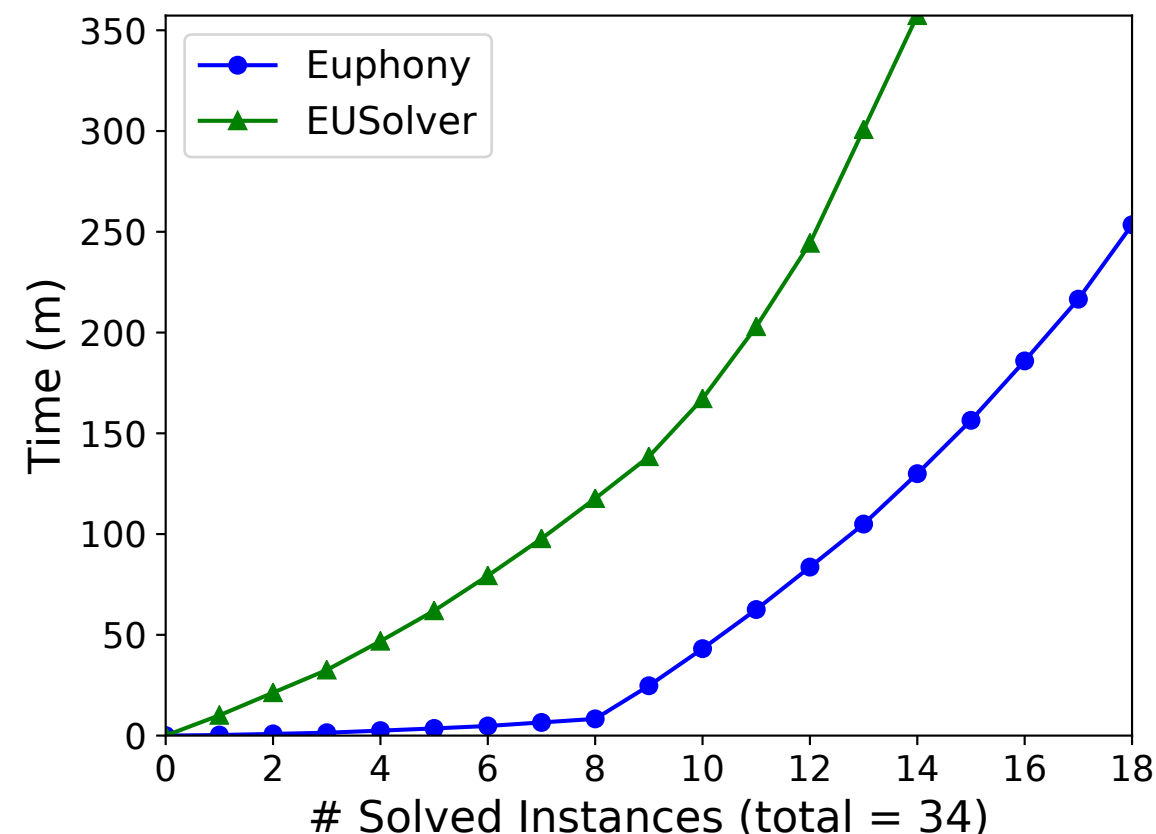
STRING



BITVEC



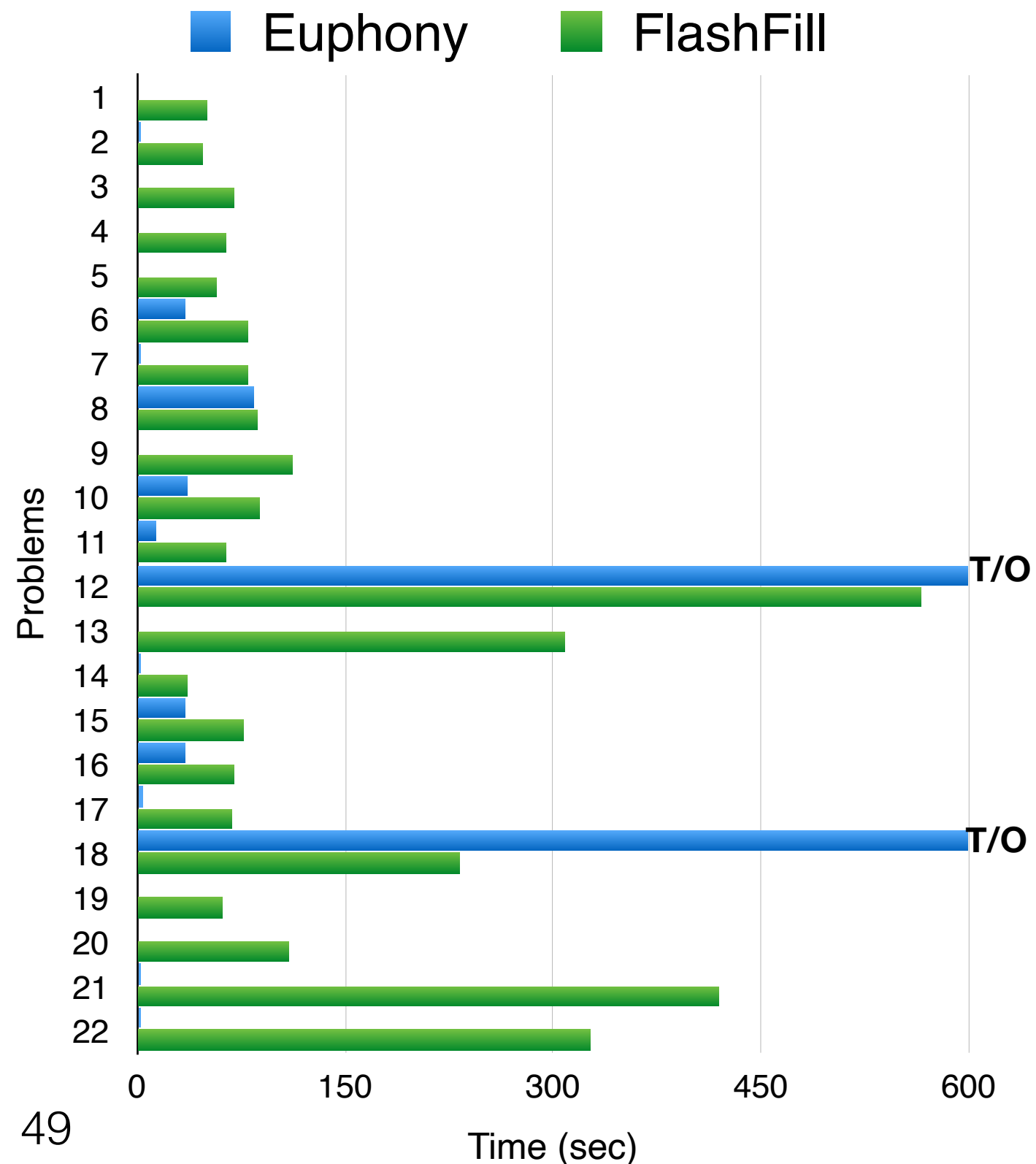
CIRCUIT



vs. FlashFill

- Training: **91** solved by FlashFill in 10 s
- Testing: **22** (timeout: 10 min)
- Euphony outperforms in **20 / 22**

	Average	Median
Euphony	13 s	3 s
Flashfill	140 s	78 s



Efficacy of A* and PHOG

- Using PCFG and PHOG [Bielik et al. ICML'16]

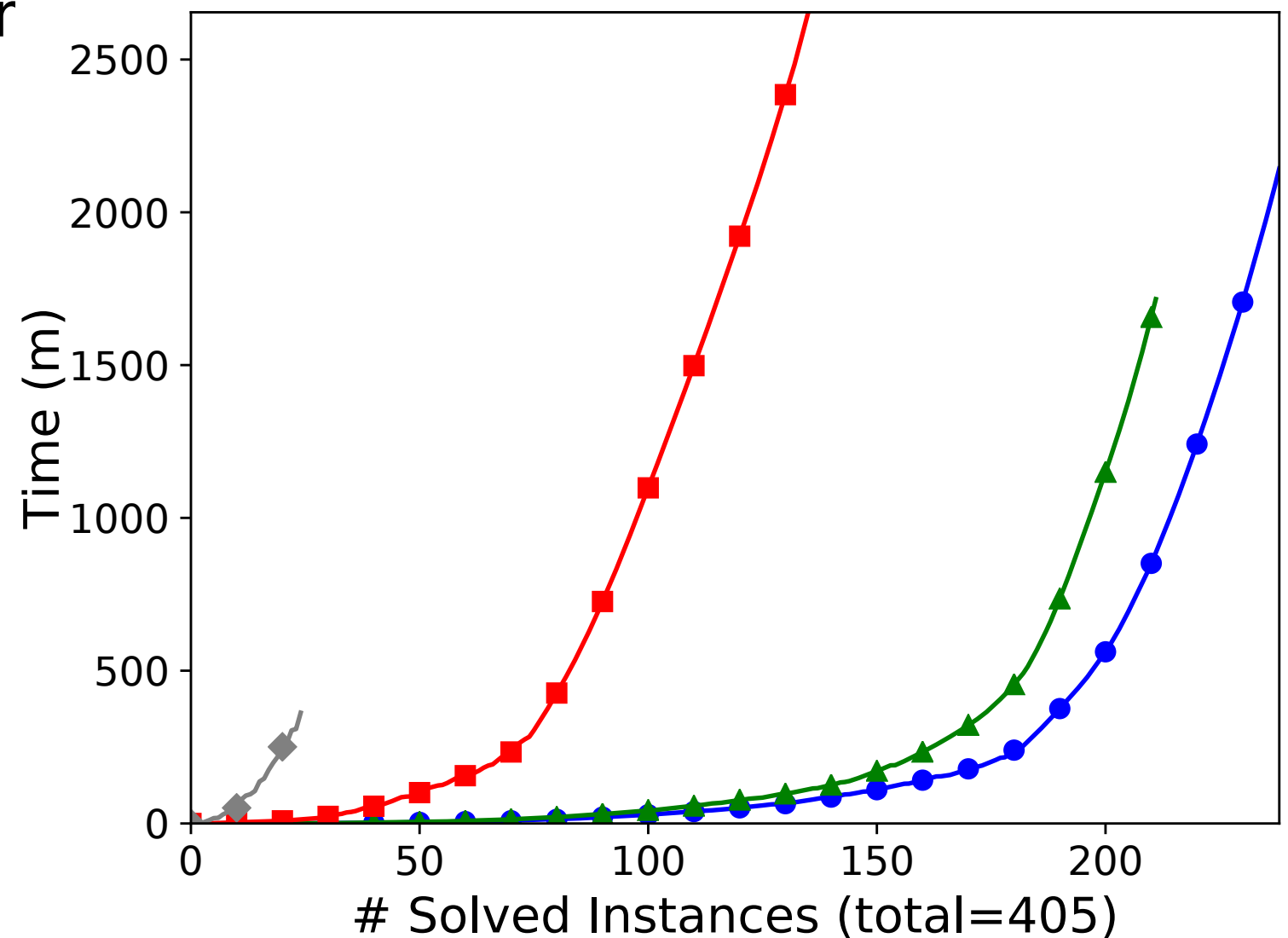
- # Solved (timeout: 1 hour)

A* + PHOG: 236

Dijkstra + PHOG: 209

A* + PCFG: 133

Dijkstra + PCFG: 22



What about Bottom-Up Search?

- Bottom-up enumeration in order of decreasing probability instead of increasing size
- Cannot consider contexts (why?)
- Use PCFGs, which are statistical models which do not consider contexts
 - Such PCFGs can be obtained from existing code, or just-in-time learning (probabilities of production rules keep changed during search).

Guided Bottom-Up Search

BottomUp(PCFG G_P , specification ϕ)

$P \leftarrow$ set of all terminals in G

cost $\leftarrow 0$

cost = $-\log$ (probability of program according to PCFG)

while True **do**

$P \leftarrow$ EnumerateExprs(G_P, P, cost)

Generate candidate programs of target cost

$P \leftarrow \{p' \in P \mid \forall p \in P. \neg \text{EQUIV}(\phi, p, p')\}$

foreach $p \in P$

if $\phi(p)$ **then return** p

cost \leftarrow cost + 1

Target cost is increased

Guided Bottom-Up Search (cont.)

EnumerateExprs(PCFG G_P, P, cost)

$P' := P$

for $N \rightarrow f(N_1, \dots, N_k) \in R$ ← For each production rule

$P' := P \cup \{f(p_1, \dots, p_k) \mid \forall i. N_i \Rightarrow^* p_i, -\log \text{Pr}(f(p_1, \dots, p_k)) = \text{cost}\}$

return P'

p_i is derivable from N_i