Homework 1
CSE6049 Program Analysis, Spring 2021
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due: 4/05(Mon), email-to-TA (bbumbuul@yahoo.com)

Exercise 1 Consider a set $T(\ni t)$ inductively defined as follows:

$$t \to \cdot \mid /t,t/ \mid /t,t,t/$$

Let $c(t)$ denote the number of occurrences of ",$''$ in t, and $s(t)$ denote the numbers of occurrences of "/$''$ in t.

Prove the following property over every $t \in T$:

$$s(t) \geq c(t)$$

□

Exercise 2 Consider the set of integer arithmetic expressions which is inductively defined as follows:

$$e \to x \mid e + e \mid e \times e \mid e \; ? \; e \; e$$

where $e_1 \; ?e_2 \; e_3$ is a conditional expression which evaluates to e_3 (resp. e_2) if e_1 evaluates to zero (resp. non-zero).

Prove the following property over every arithmetic expresesion e: if every variable that appears in e holds a multiple of n, the evaluation result of e is also a multiple of n. For example, if $x = 4$ and $y = 2$ (both variables hold a multiple of 2), $x + y$ evaluates to 6 which is also a multiple of 2. □

Exercise 3 Find the least fixpoint for each of the following functions.

- $\lambda x.\ 1 \quad \in \mathbb{Z} \to \mathbb{Z}$
- $\lambda x.\ x \quad \in \mathbb{Z} \to \mathbb{Z}$
- $\lambda x.\ x + 1 \quad \in \mathbb{Z} \cup \{\infty\} \to \mathbb{Z} \cup \{\infty\}$

- $\lambda f.\ (\lambda x.\ if\ x = 0\ then\ 0\ else\ x + f(x-1)) \quad \in (\mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N})$
- $\lambda X.\ \{\epsilon\} \cup \{ax \mid x \in X\} \quad \in 2^S \to 2^S$ where S is the set of finite strings and 2^A denotes the powerset of A for set A.

□

Exercise 4 Prove the following:
Given two CPOs $(D_1, \sqsubseteq_1)$ and $(D_2, \sqsubseteq_2)$, $(D, \sqsubseteq)$ is a CPO where

$$D = D_1 \times D_2 = \{(d_1, d_2) \mid d_1 \in D_1, d_2 \in D_2\}$$

and

$$(d_1, d_2) \sqsubseteq (d_1', d_2') \iff (d_1 \sqsubseteq_1 d_1') \wedge (d_2 \sqsubseteq_1 d_2').$$

□