Optimizing Homomorphic Evaluation Circuit with Program Synthesis and Term Rewriting

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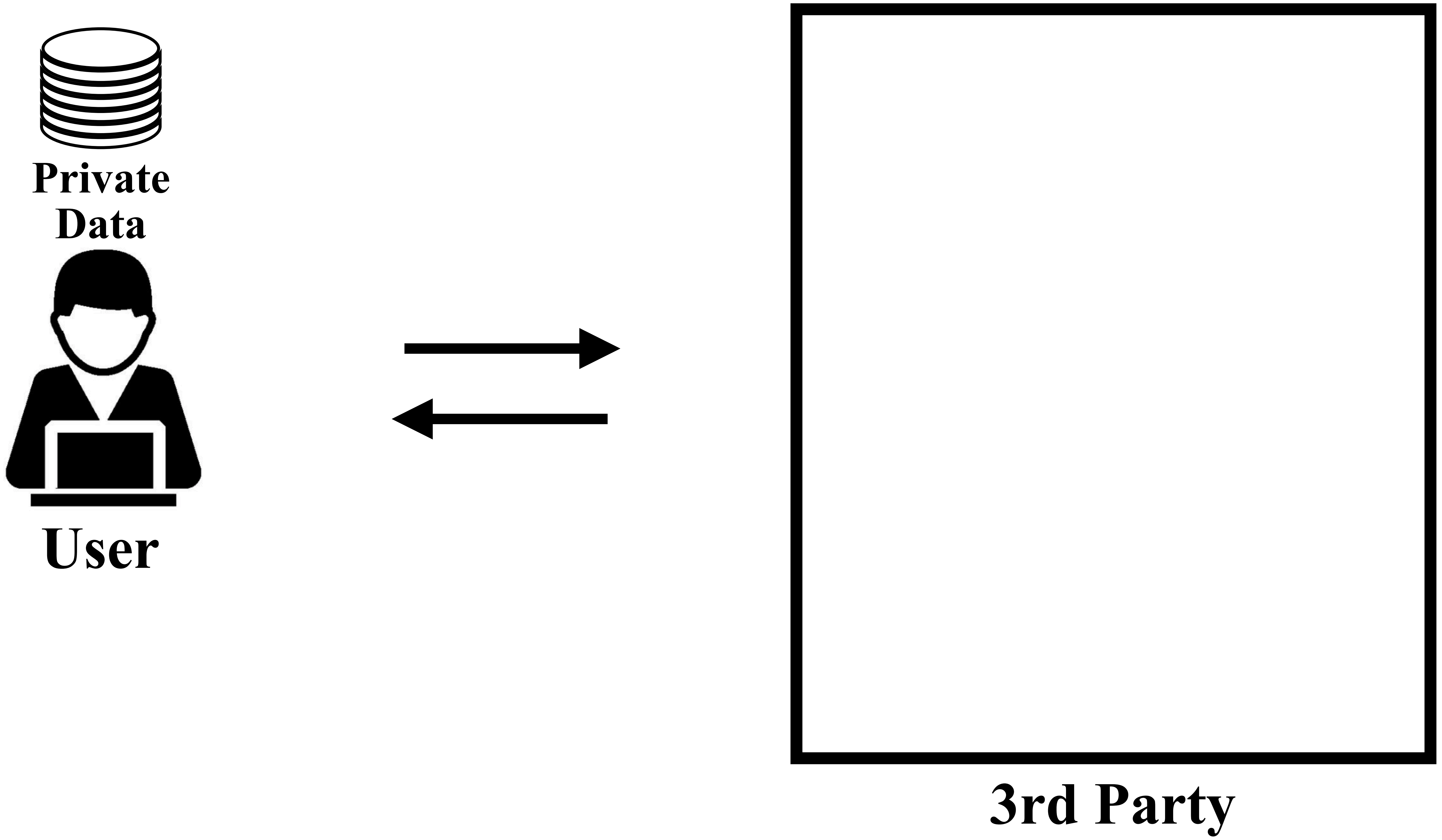
Homomorphic Evaluation(HE) (1/3)

Privacy Preserving Secure Computation

- Allows for computation on encrypted data
- Enables the outsourcing of private data storage/processing



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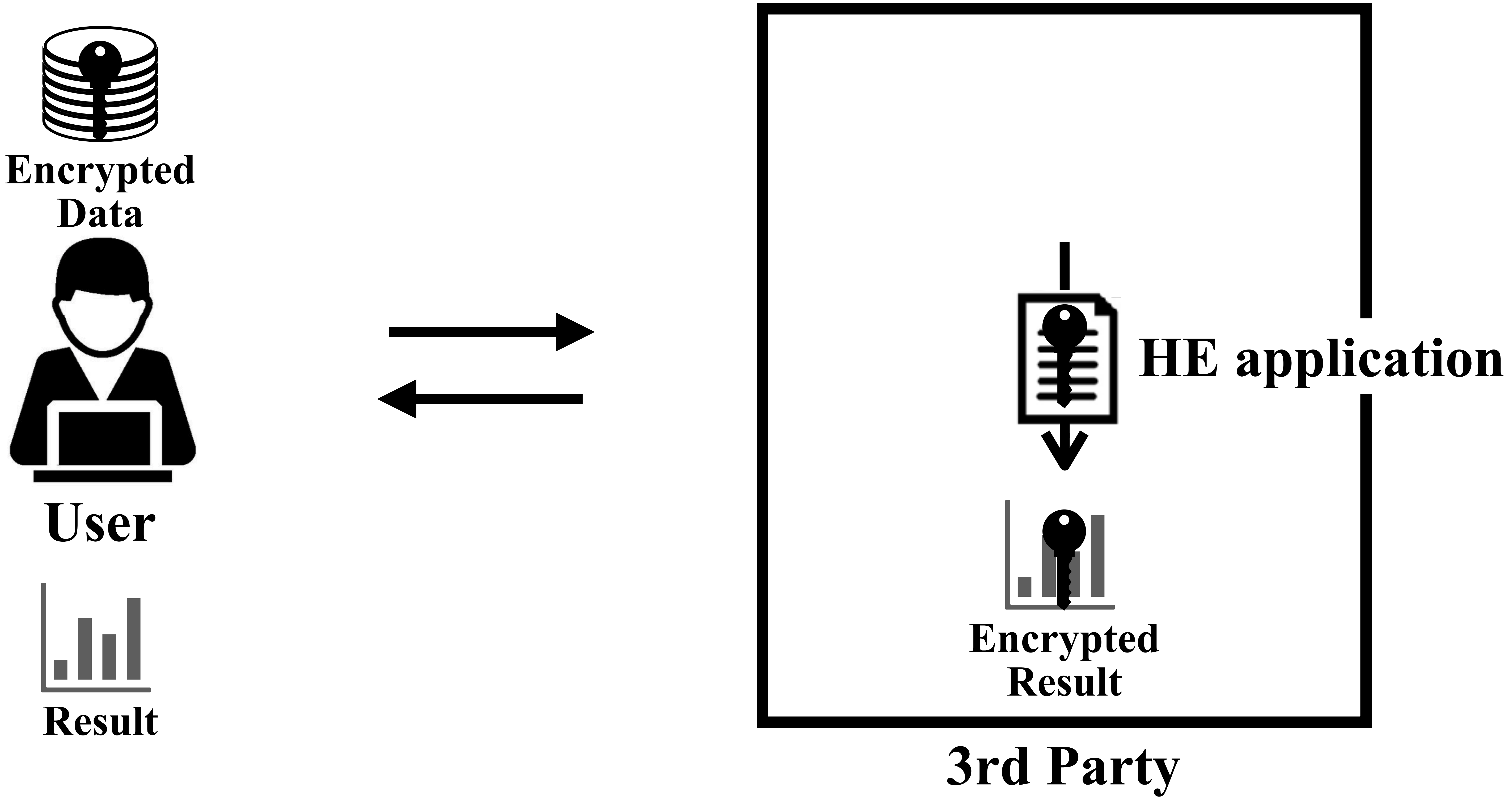
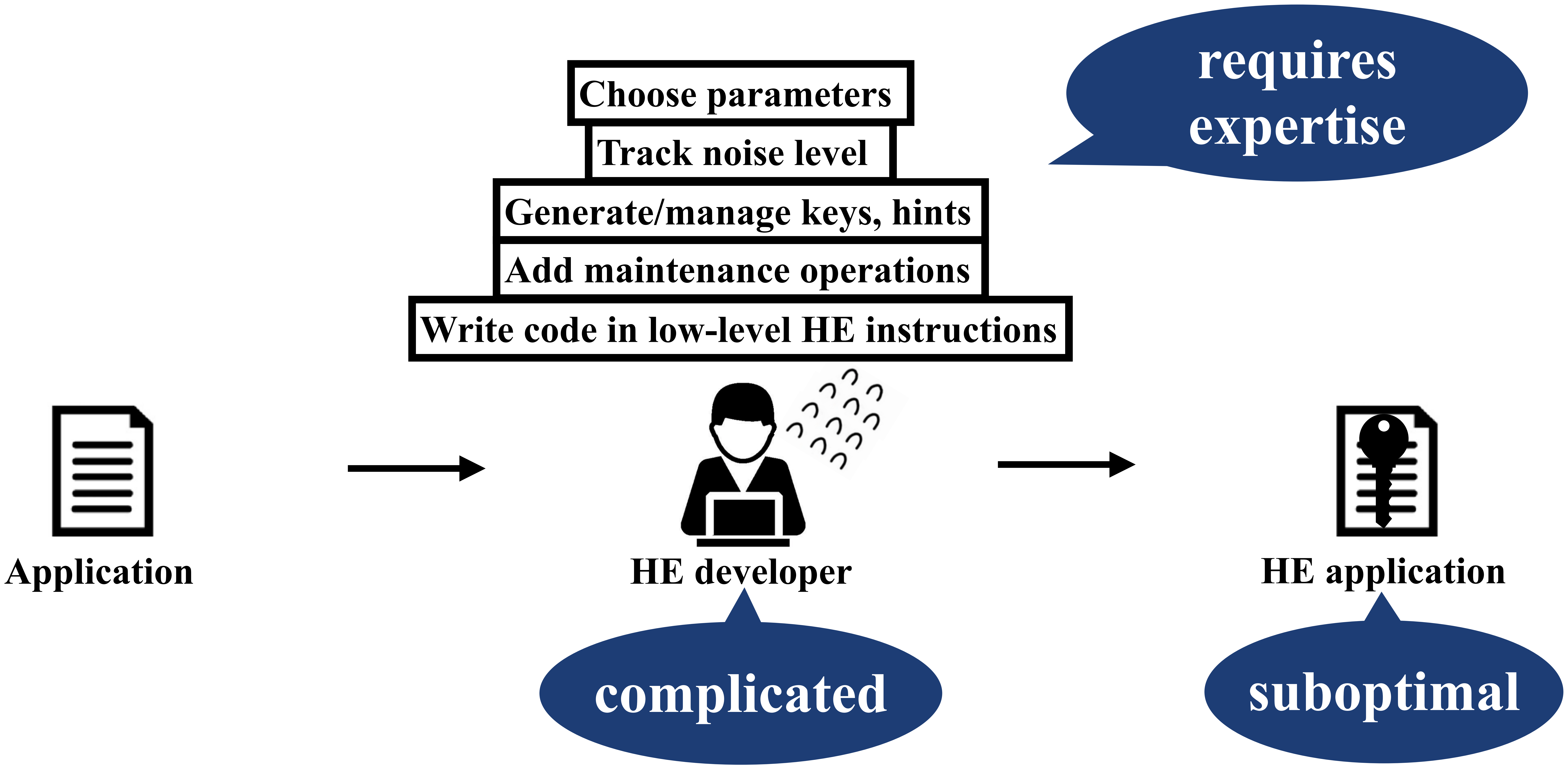


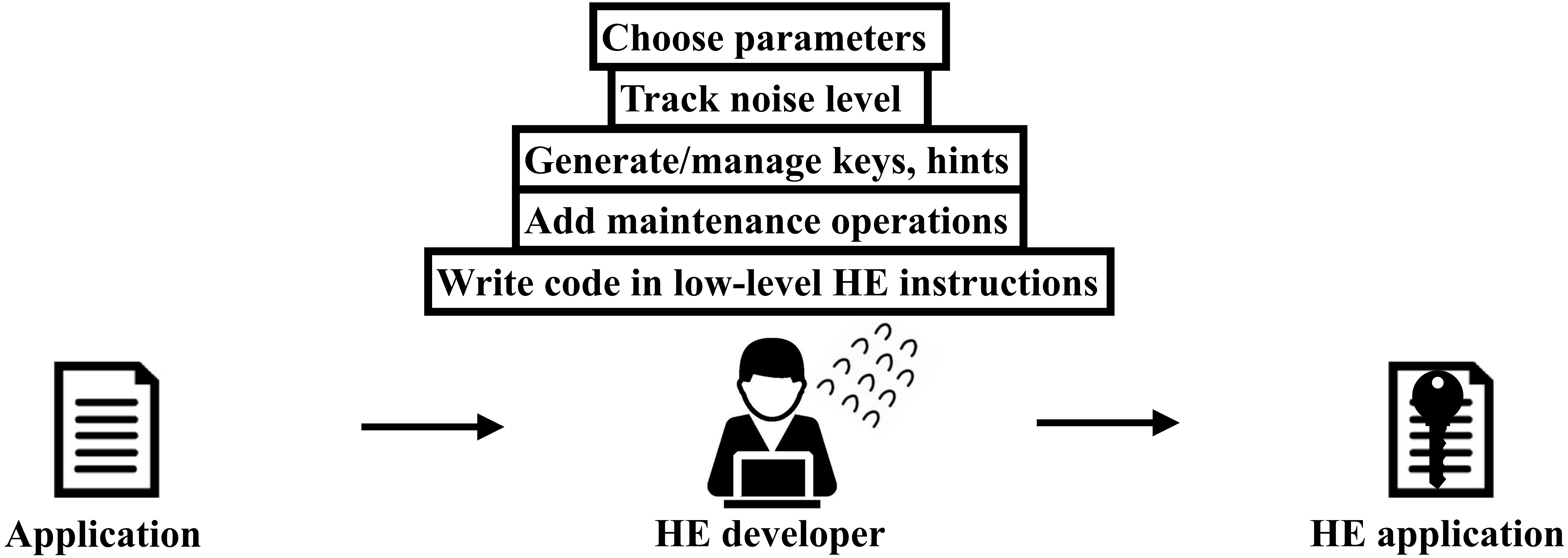
Homomorphic Evaluation(HE) (2/3)

Building HE applications



Homomorphic Evaluation(HE) (3/3)

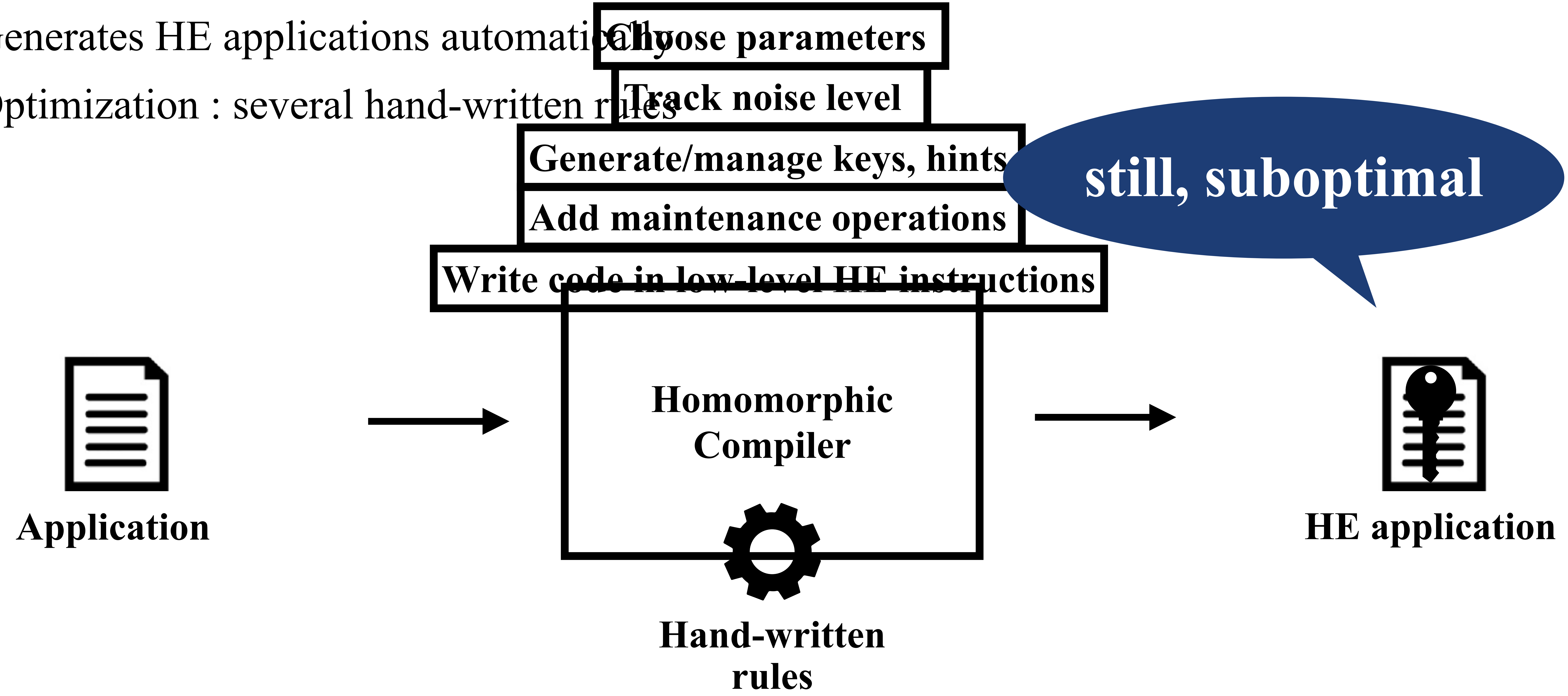
Existing Homomorphic Compiler



Homomorphic Evaluation(HE) (3/3)

Existing Homomorphic Compiler

- Generates HE applications automatically
- Optimization : several hand-written rules



Homomorphic Evaluation(HE) (2/3)

Code for homomorphic addition of two integers

```
#include "FHE.h"
#include "EncryptedArray.h"
#include <NTL/lzz_pXFactoring.h>
#include <fstream>
#include <sstream>
#include <sys/time.h>

int main(int argc, char **argv)
{
    long m=0, p=2, r=1; // Native plaintext space
                        // Computations will be 'modulo p'
    long L=16;          // Levels
    long c=3;           // Columns in key switching matrix
    long w=64;          // Hamming weight of secret key
    long d=0;
    long security = 128;
    ZZX G;
    m = FindM(security,L,c,p, d, 0, 0);
    FHEcontext context(m, p, r);
    buildModChain(context, L, c);
    FHESecKey secretKey(context);
    const FHEPubKey& publicKey = secretKey;
    G = context.alMod.getFactorsOverZZ()[0];
    secretKey.GenSecKey(w);
    addSome1DMatrices(secretKey);
    EncryptedArray ea(context, G);
    vector<long> v1;
    v1.push_back(atoi(argv[1]));
    Ctxt ct1(publicKey);
    ea.encrypt(ct1, publicKey, v1);
    v2.push_back(atoi(argv[2]));
    Ctxt ct2(publicKey);
    ea.encrypt(ct2, publicKey, v2);
    Ctxt ctSum = ct1;
    ctSum += ct2;
}
```

Manually written using HElib

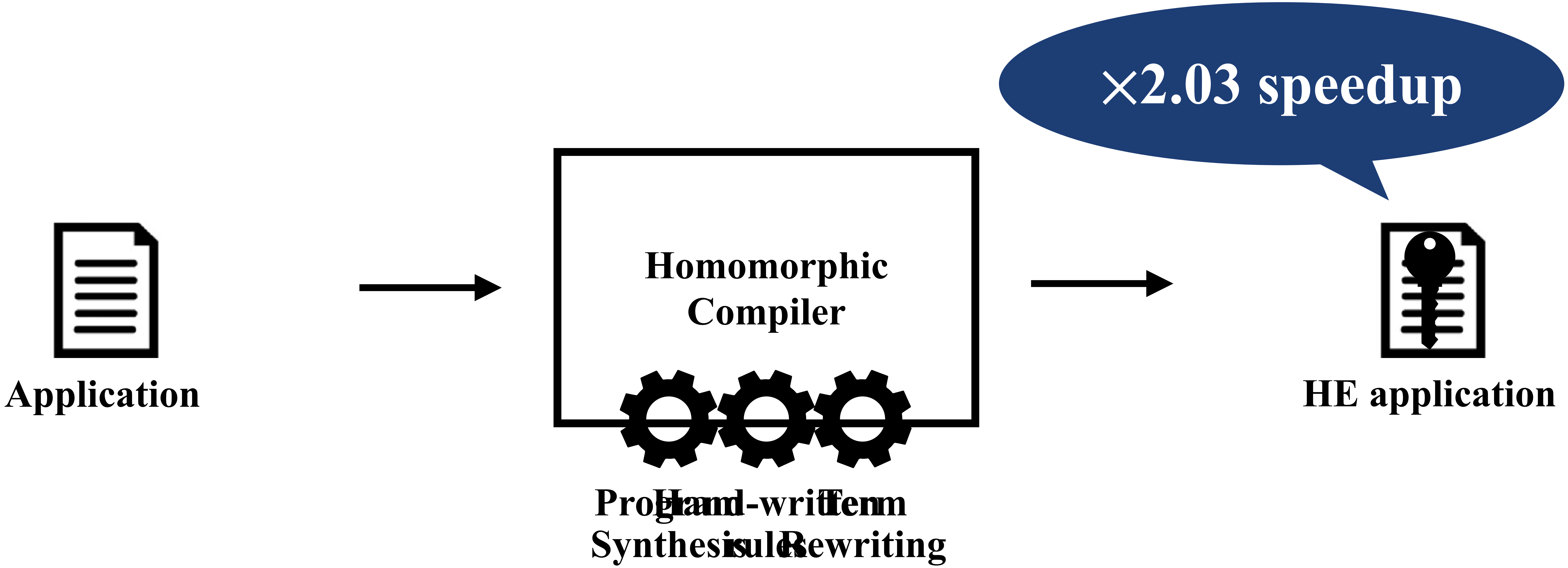
```
#include <iostream>
#include <fstream>
#include <integer.hxx>

int main()
{
        Integer8 a, b, c;

        cin >> a;
        cin >> b;
        c = a + b;

        cout << c;
        FINALIZE_CIRCUIT(blif_name);
}
```

Input to Cingulata (a HE compiler)

Our Contributions (1/2)

Automatic, Aggressive HE optimization Framework

- Generates HE applications automatically
- Optimization : search hand-written rules
- Optimization : new rules found by program synthesis + applying by term rewriting



Our Contributions (2/2)

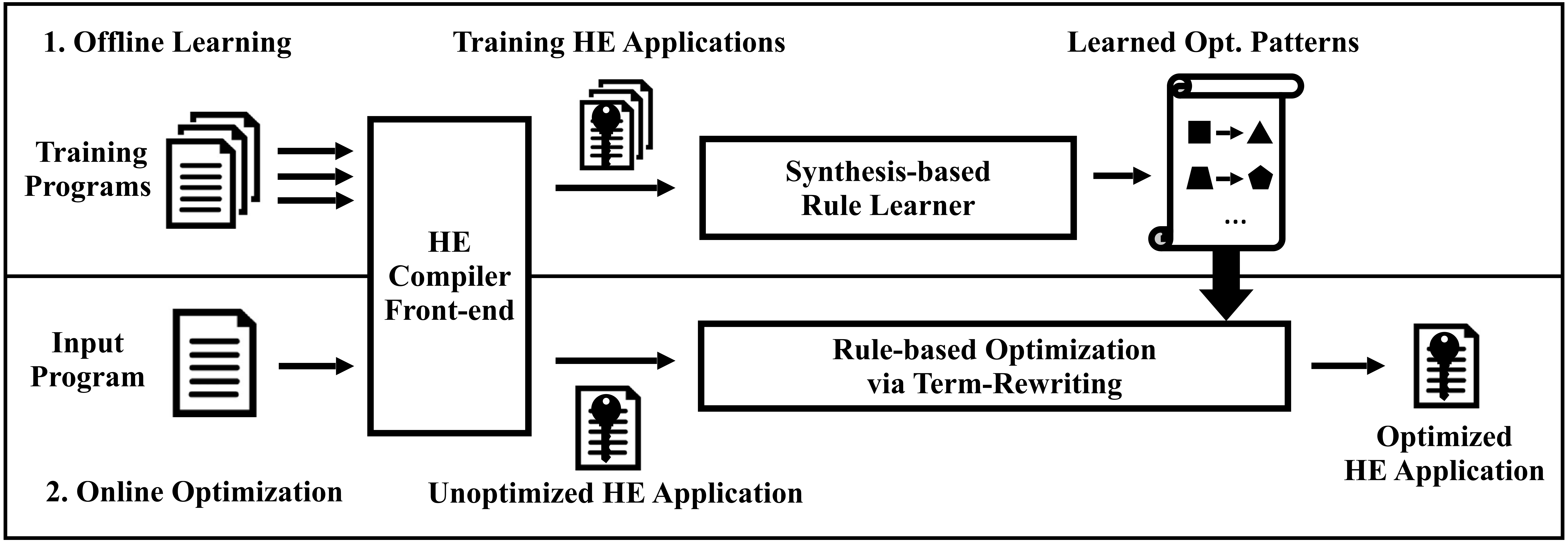
Automatic, Aggressive HE optimization Framework

- Learning Optimization Patterns by Program Synthesis
- Applying Learned Patterns by Term Rewriting
- Theorem : Semantic Preservation & Termination Guaranteed
- Performance (vs state-of-the-art HE Optimizer)
  - Optimized 19 out of 25 Applications (vs 15)
  - x3.71 Speedup in Maximum (vs x3.0)
  - x2.03 Speedup on Average (vs x1.53)
- Open Tool Available : https://github.com/dklee0501/Lobster

Our Lobster 

Learning to Optimize Boolean circuit using Synthesis and TErm Rewriting

- Offline Learning via Program Synthesis + Online Optimization via Term Rewriting



Simple HE Scheme

- Based on approximate common divisor problem
- p : integer as a secret key
- q : random integer
- r ($\ll |p|$) : random noise for security

$$Enc_p(\ \mu \in \{0,1\}\ ) = pq + 2r + \mu$$

$$Dec_p(c) = (c \bmod p) \bmod 2$$

$$Dec_p(Enc_p(\mu)) = Dec_p(\cancel{pq} + \cancel{2r} + \mu) = \mu$$

- For ciphertexts $\underline{\mu_i} \leftarrow Enc_p(\mu_i)$, the following holds

$$Dec_p(\underline{\mu_1} + \underline{\mu_2}) = \mu_1 + \mu_2$$
$$Dec_p(\underline{\mu_1} \times \underline{\mu_2}) = \mu_1 \times \mu_2$$

- The scheme can evaluate all boolean circuits as $+$ and $\times$ in $\mathbb{Z}_2 = \{0,1\}$ are equal to XOR and AND

Performance Hurdle : Growing Noise

- Noise increases during homomorphic operations.
- For $\underline{\mu_i} = pq_i + 2r_i + \mu_i$

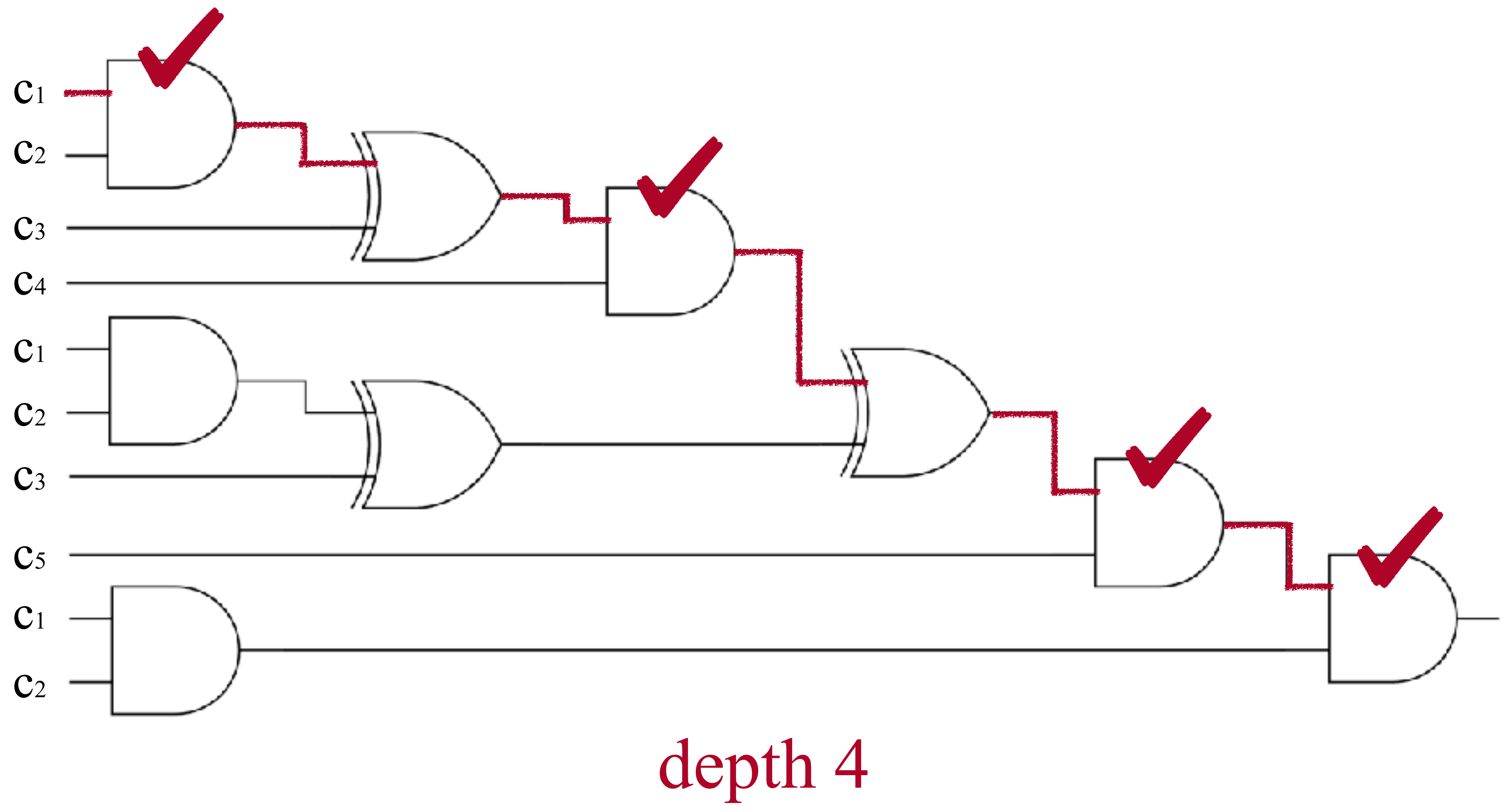
$$\begin{aligned}
\underline{\mu_1} + \underline{\mu_2} &= p(q_1 + q_2) + \boxed{2(r_1 + r_2) + (\mu_1 + \mu_2)} \text{ double increase} \\
\underline{\mu_1} \times \underline{\mu_2} &= p(pq_1q_2 + \cdots) + \boxed{2(2r_1r_2 + r_1\mu_2 + r_2\mu_1) + (\mu_1 \times \mu_2)} \text{ quadratic increase}
\end{aligned}$$

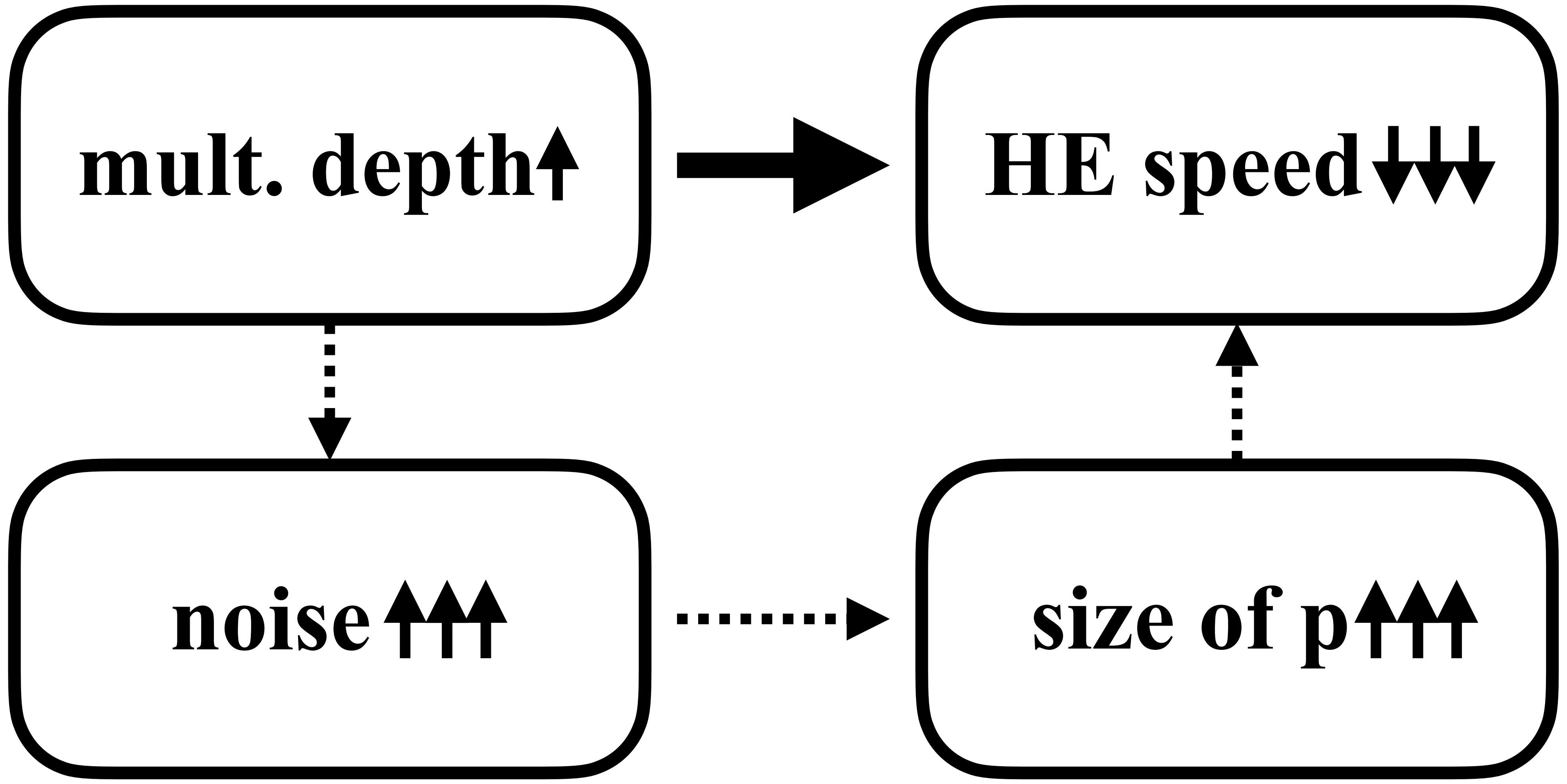
noise

- **if (noise $>$ p) then incorrect results**

Multiplicative Depth : a Decisive Performance Factor

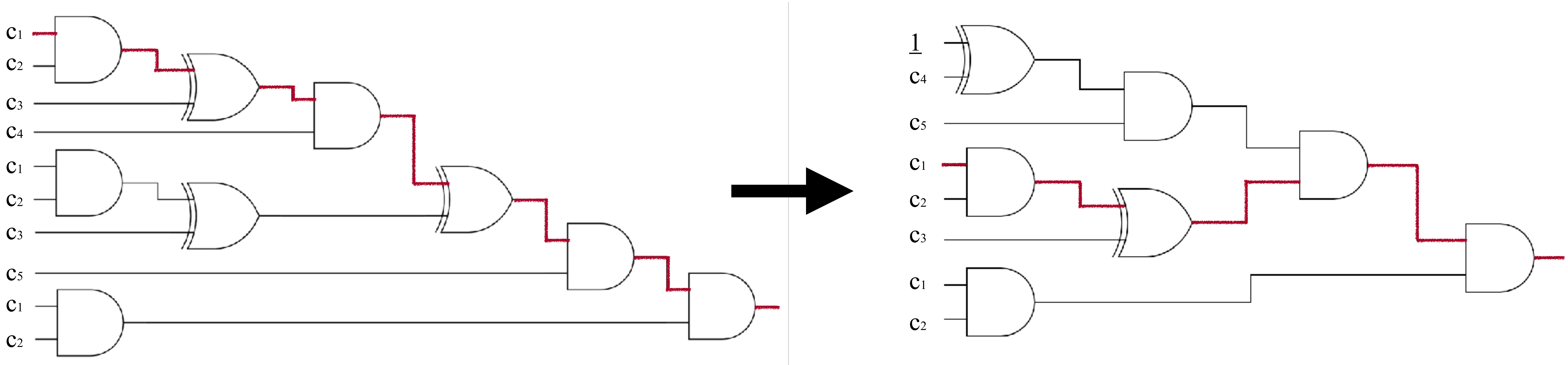
- Multiplicative depth : the maximum number of sequential multiplications from input to output





What is HE optimization?

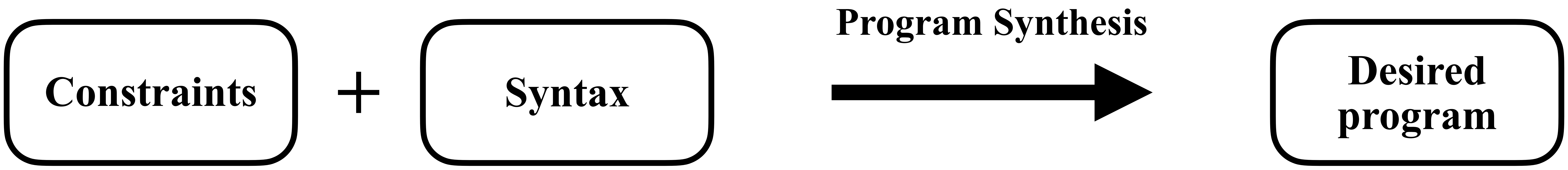
- Finding a new circuit that has smaller mult. depth



depth 4

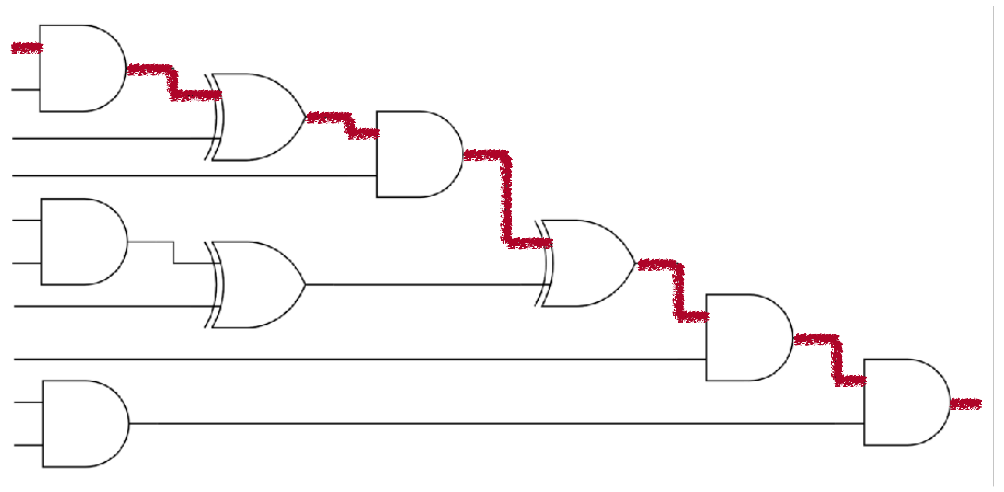
depth 3

HE optimization via Synthesis



HE optimization via Synthesis

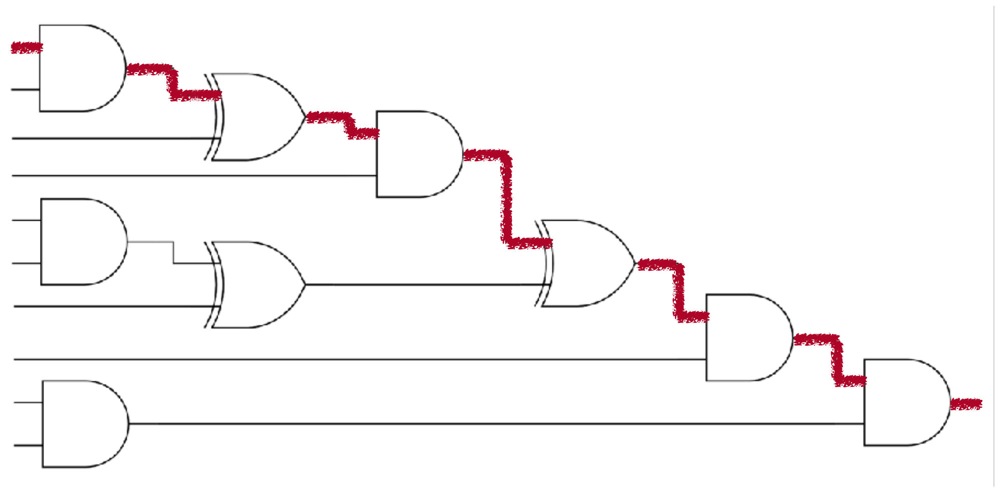
Constraints + Syntax —Program Synthesis→ Desired program



same semantics

HE optimization via Synthesis

Constraints + **Syntax** $\xrightarrow{\text{Program Synthesis}}$ **Desired program**



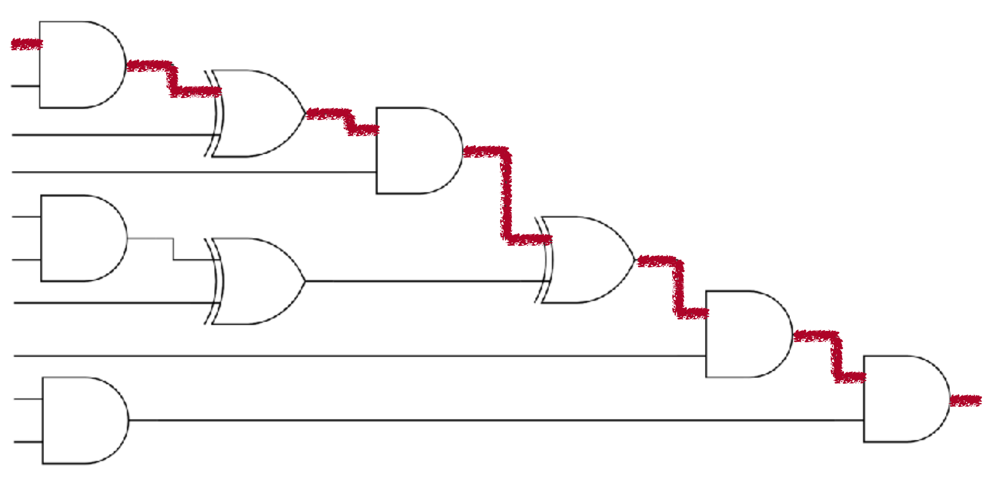
same semantics

$$
\begin{aligned}
S &\rightarrow d_3 \\
d_3 &\rightarrow d_2 \wedge d_2 \mid d_3 \oplus d_3 \mid d_2 \\
d_2 &\rightarrow d_1 \wedge d_1 \mid d_2 \oplus d_2 \mid d_1 \\
d_1 &\rightarrow d_0 \wedge d_0 \mid d_1 \oplus d_1 \mid d_0 \\
d_0 &\rightarrow 0 \mid 1 \mid c_1 \mid c_2 \mid c_3 \mid c_4 \mid c_5
\end{aligned}
$$

depth-restricting syntax

HE optimization via Synthesis

Constraints + **Syntax** —Program Synthesis→ **Desired program**



same semantics

$$
\begin{array}{lcl}
S & \rightarrow & d_3 \\
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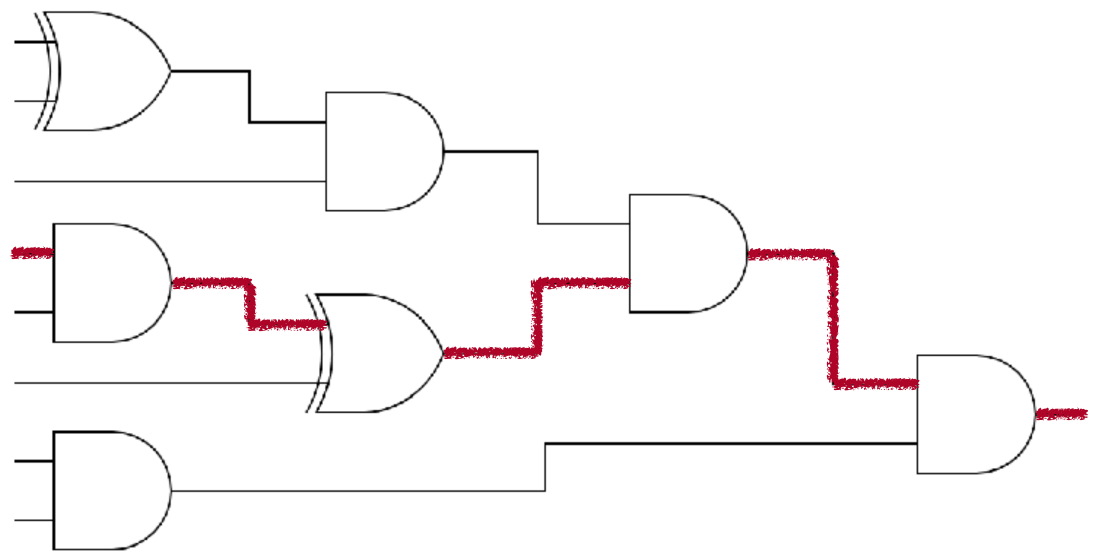
d_2 & \rightarrow & d_1 \wedge d_1 \mid d_2 \oplus d_2 \mid d_1 \\
d_1 & \rightarrow & d_0 \wedge d_0 \mid d_1 \oplus d_1 \mid d_0 \\
d_0 & \rightarrow & 0 \mid 1 \mid c_1 \mid c_2 \mid c_3 \mid c_4 \mid c_5
\end{array}
$$

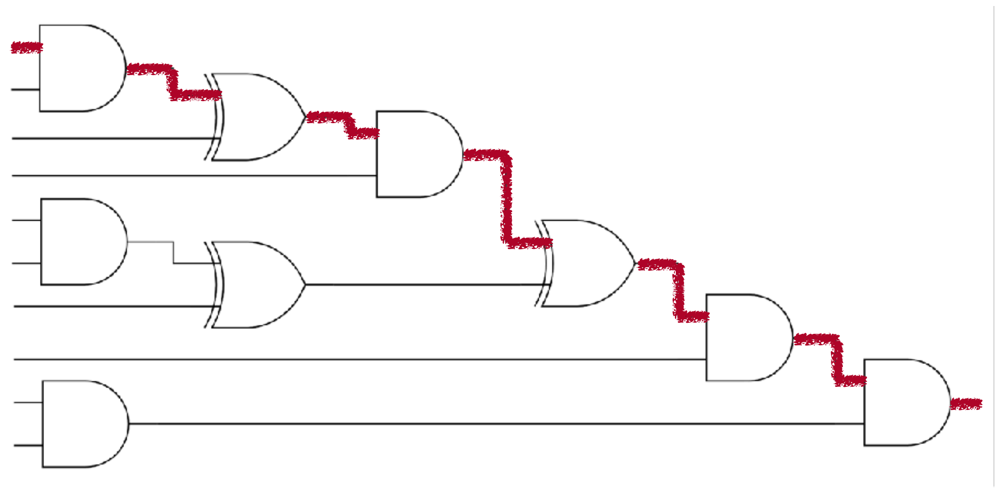
depth-restricting syntax



optimized HE circuit

HE optimization via Synthesis

Constraints + **Syntax** $\xrightarrow{\text{Optimizing Synthesis}}$ **Desired program**

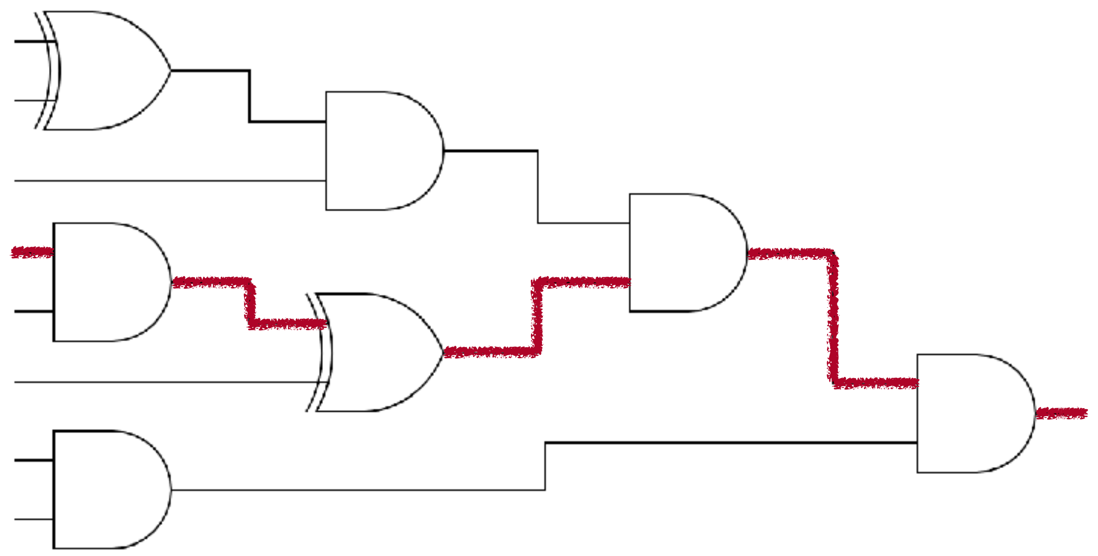


depth 4

same semantics

$$\begin{aligned} S &\rightarrow d_3 \\ d_3 &\rightarrow d_2 \wedge d_2 \mid d_3 \oplus d_3 \mid d_2 \\ d_2 &\rightarrow d_1 \wedge d_1 \mid d_2 \oplus d_2 \mid d_1 \\ d_1 &\rightarrow d_0 \wedge d_0 \mid d_1 \oplus d_1 \mid d_0 \\ d_0 &\rightarrow 0 \mid 1 \mid c_1 \mid c_2 \mid c_3 \mid c_4 \mid c_5 \end{aligned}$$

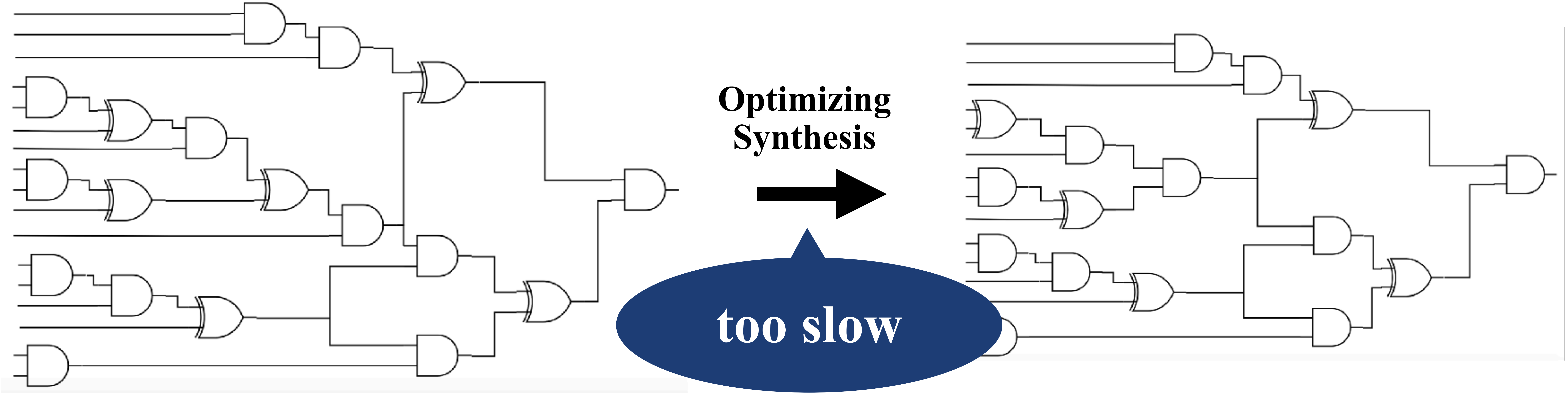
depth-restricting syntax



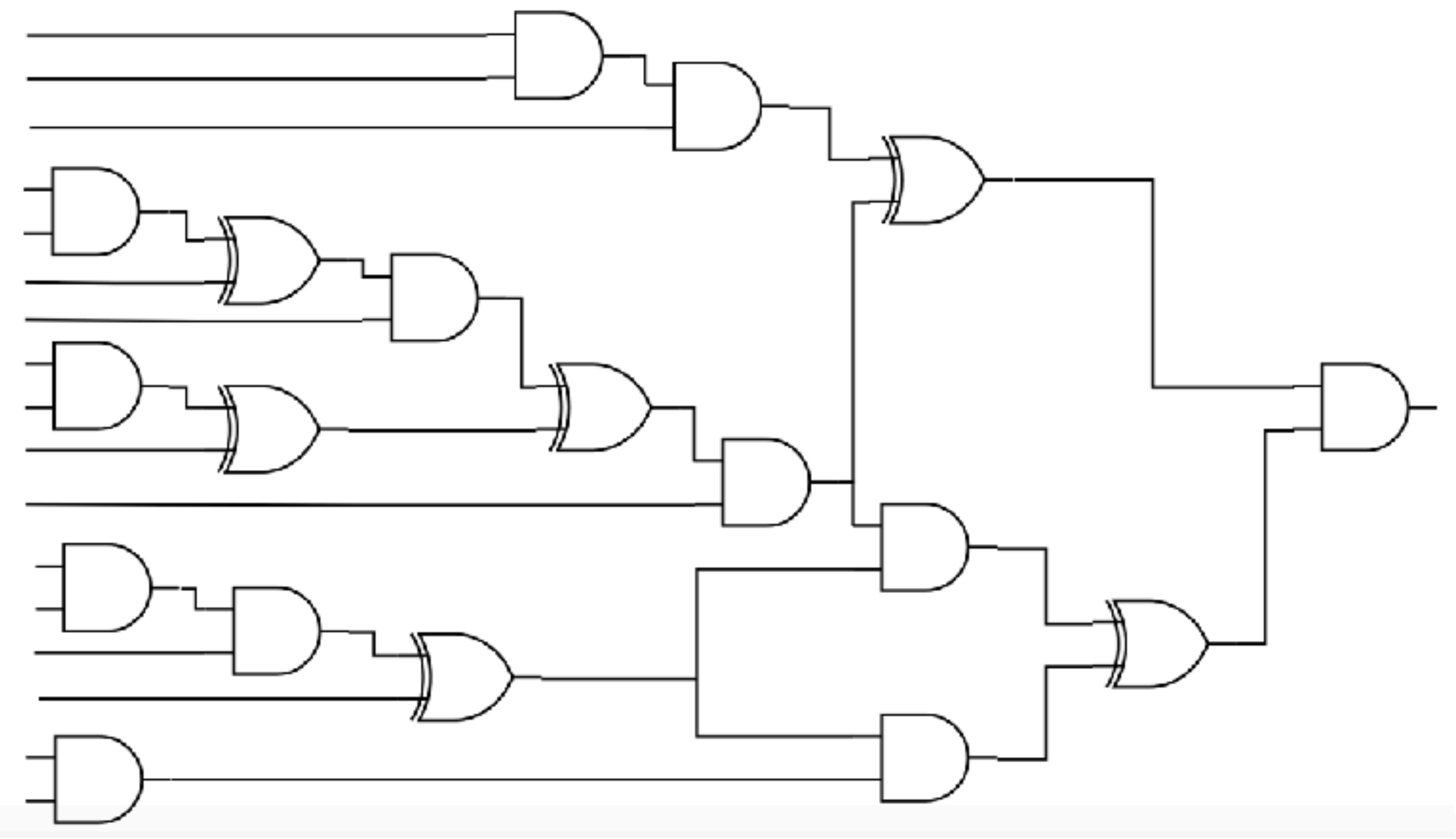
depth 3

optimized HE circuit

Hurdle : Synthesis Scalability



Solution1 : Synthesis via Localization



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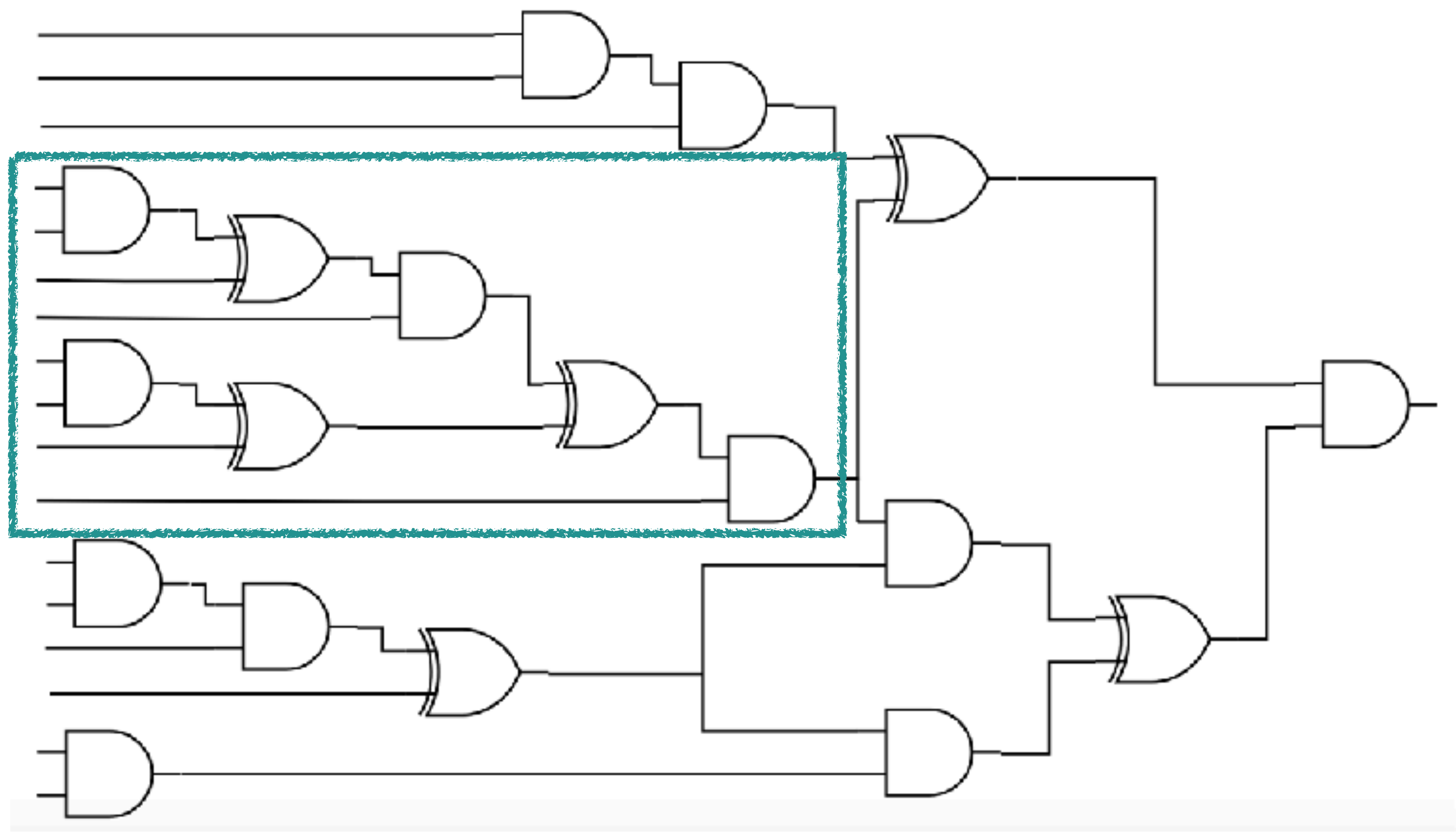

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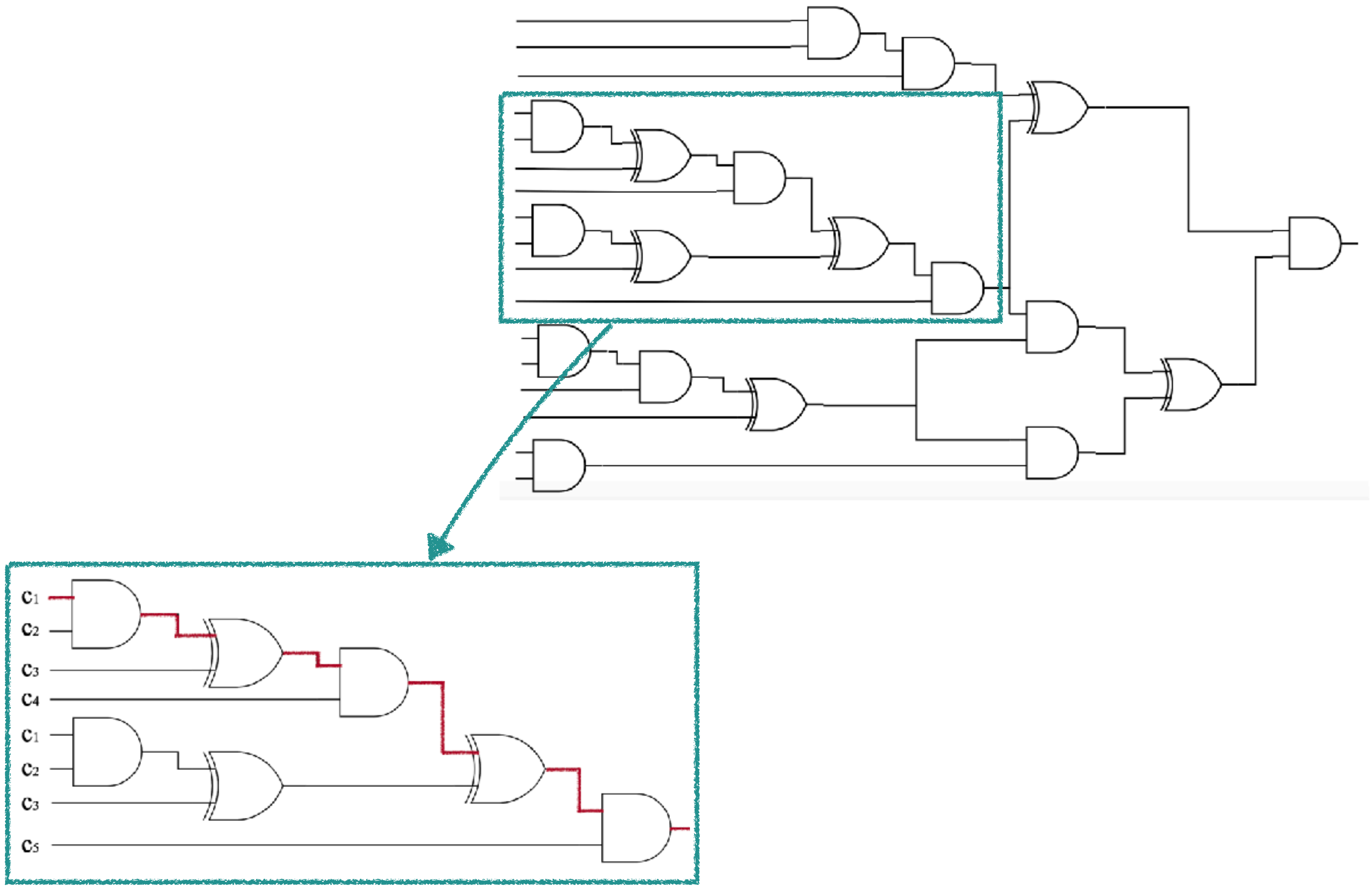

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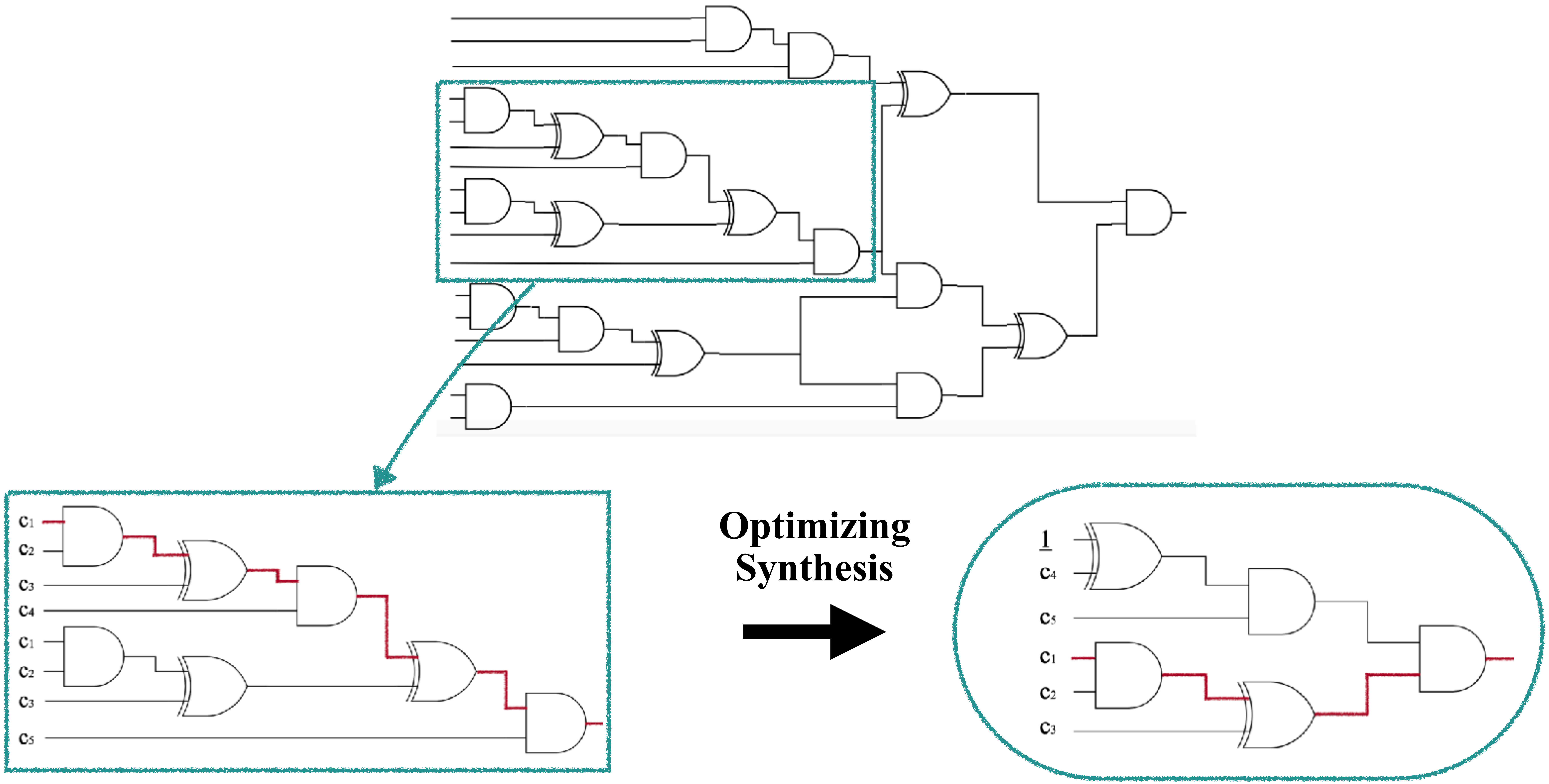

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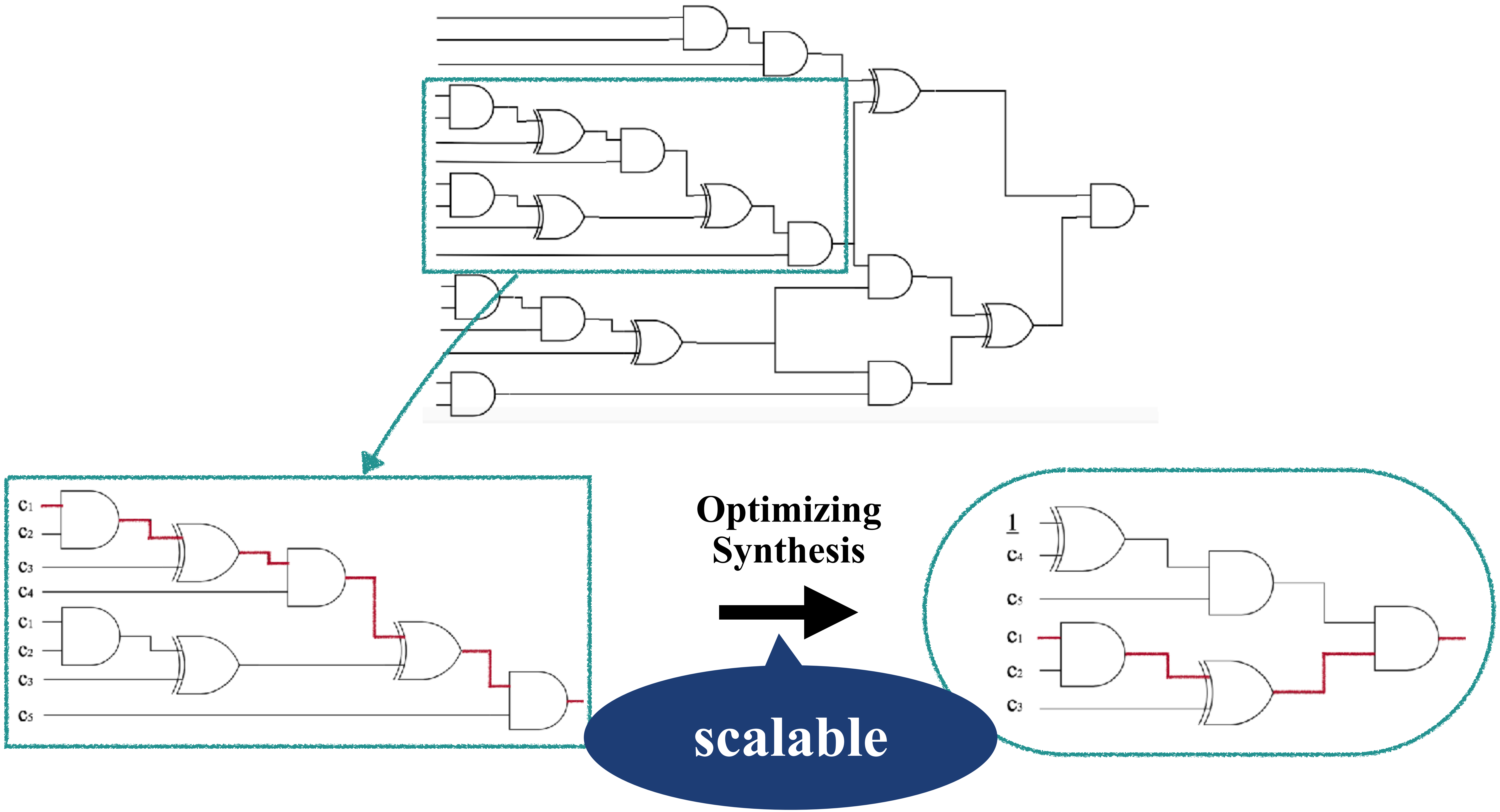

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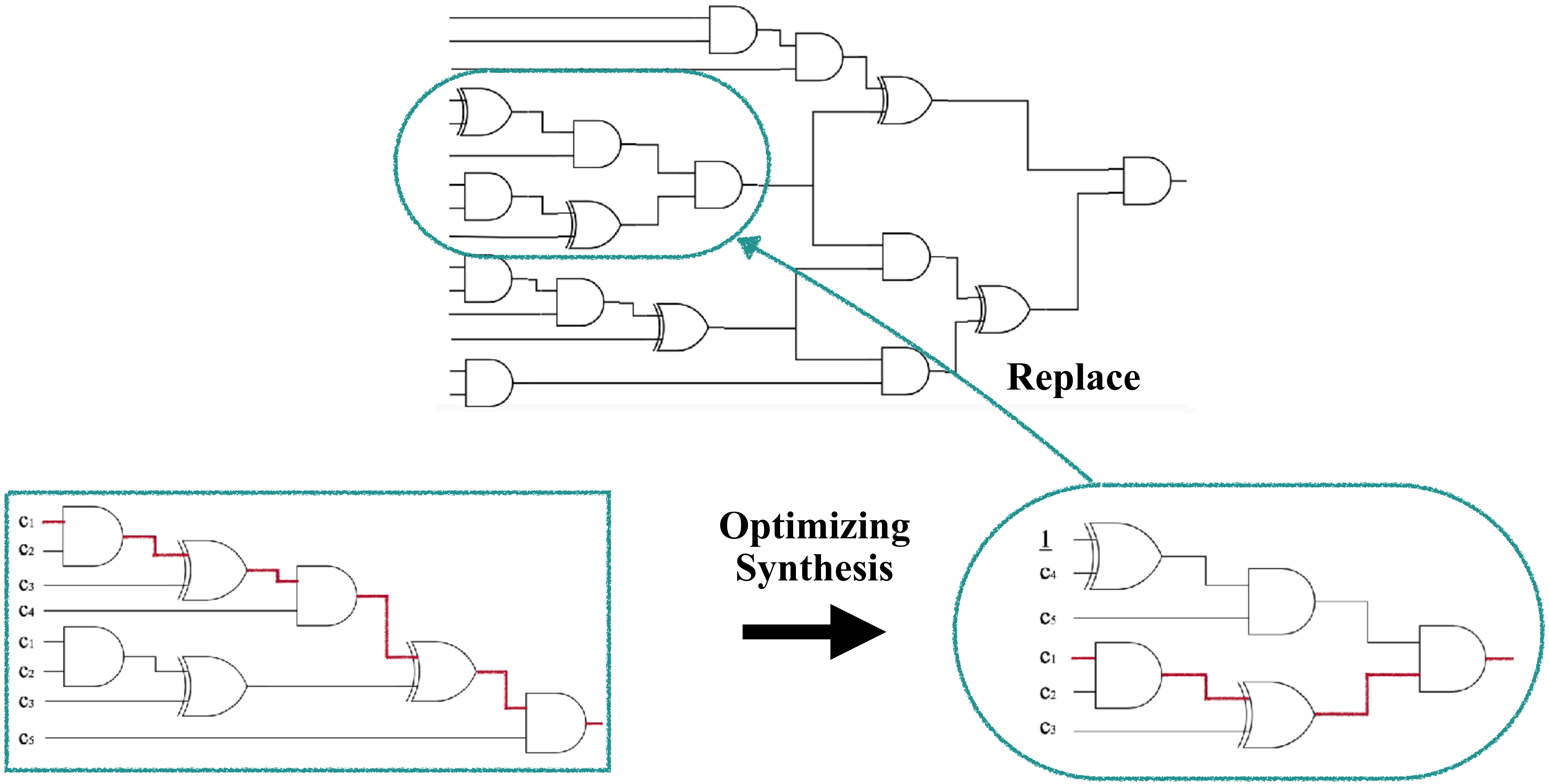
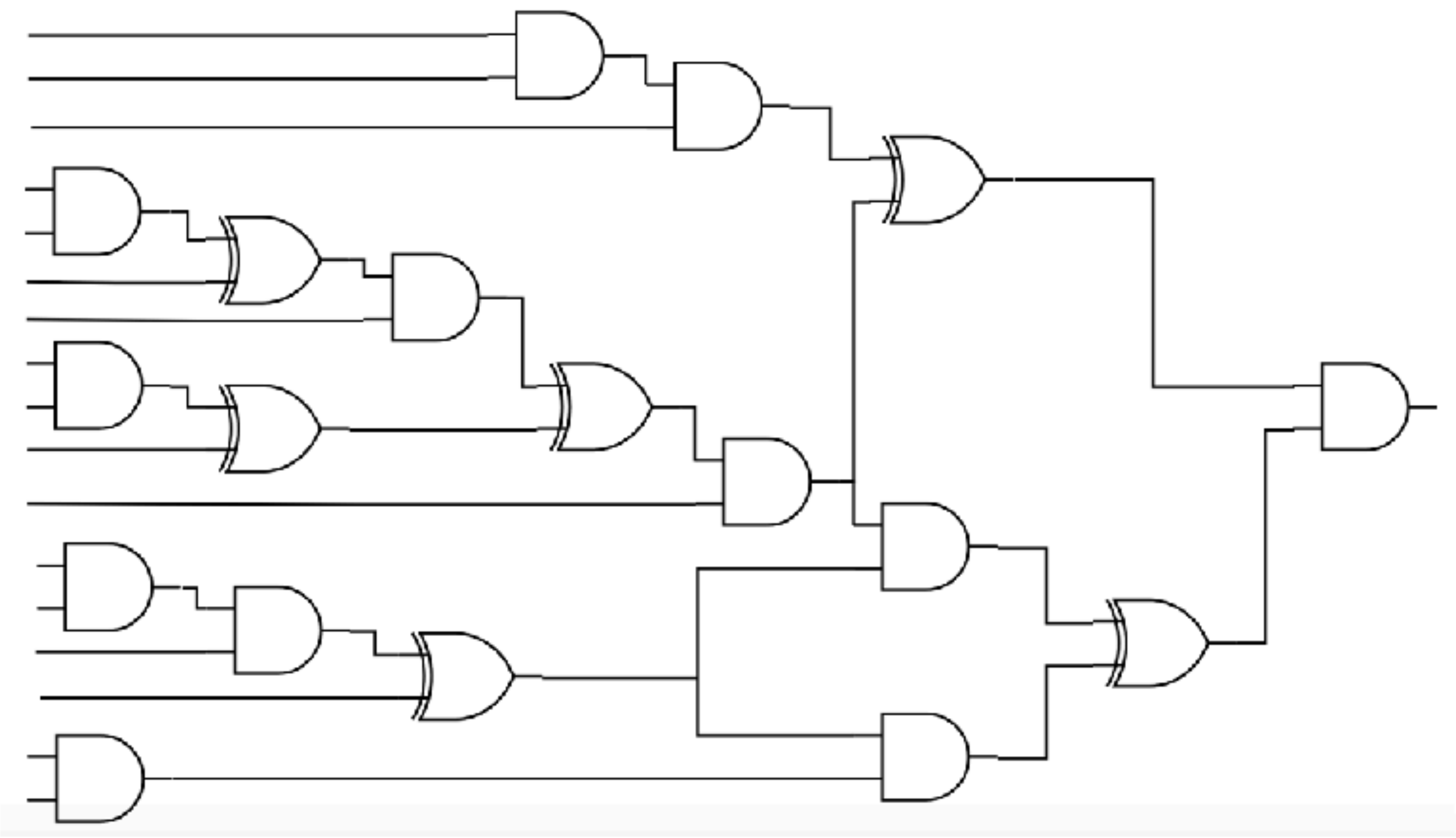


Solution 2: Learning Successful Synthesis Patterns

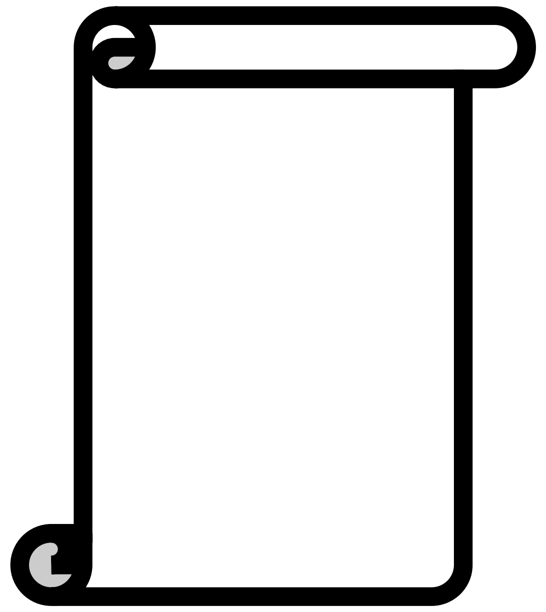
- Offline Learning
  - Collect successful synthesis patterns
- Online Optimization
  - Applying the patterns by term rewriting

Offline Learning to Collect Opt. Patterns

Training HE Applications

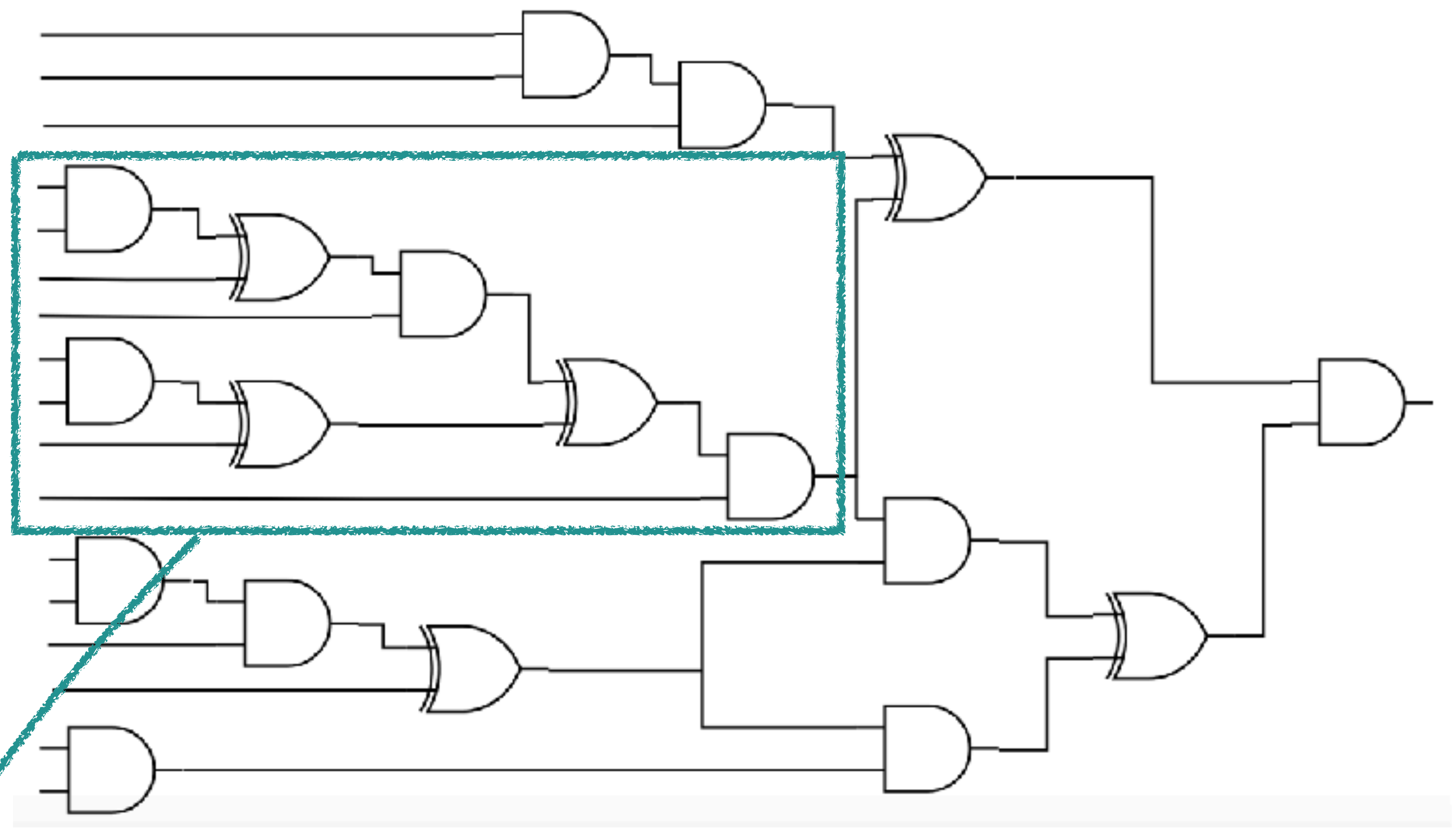


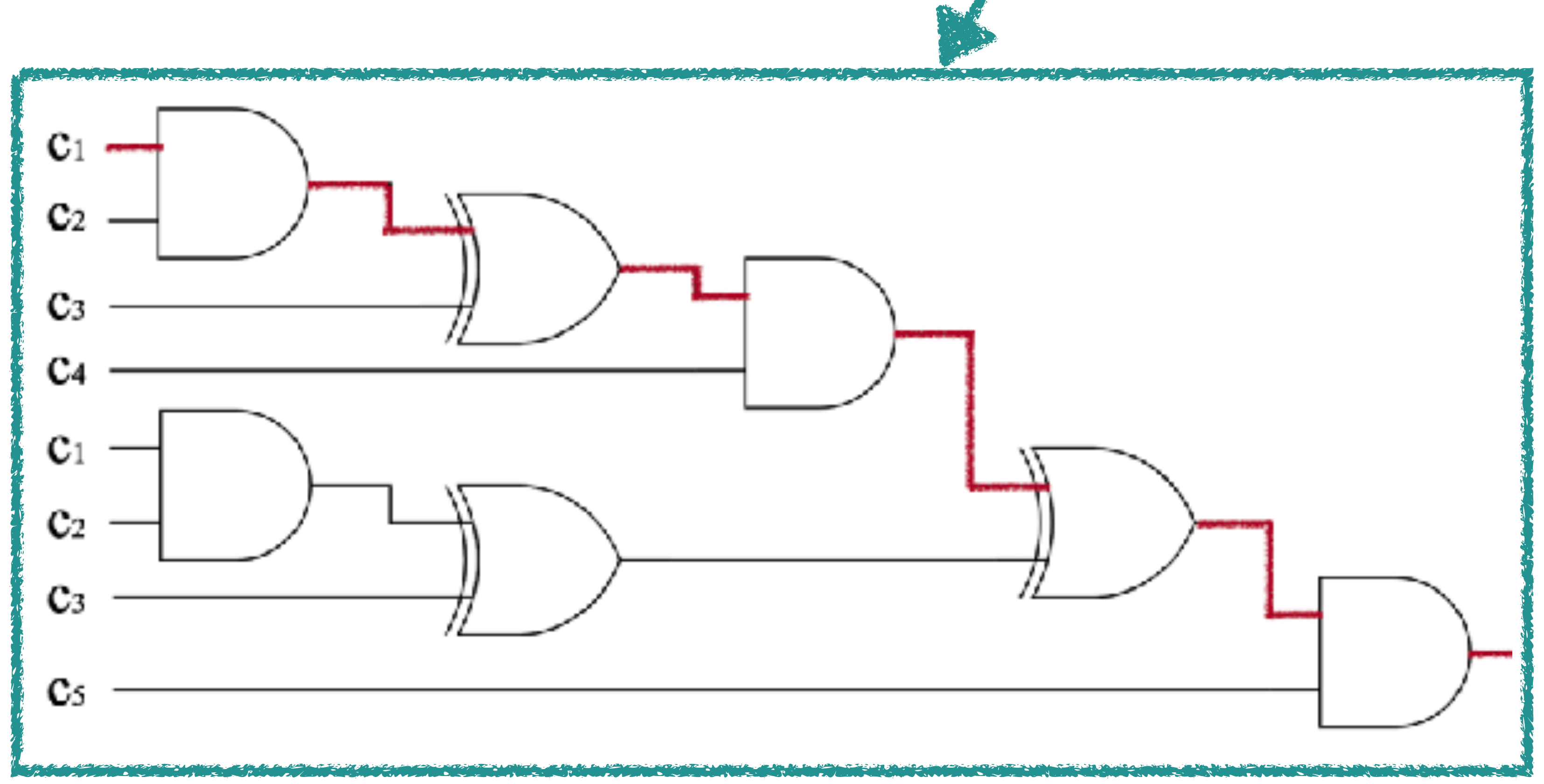
Collected Opt. Patterns



Offline Learning to Collect Opt. Patterns

Training
HE Applications





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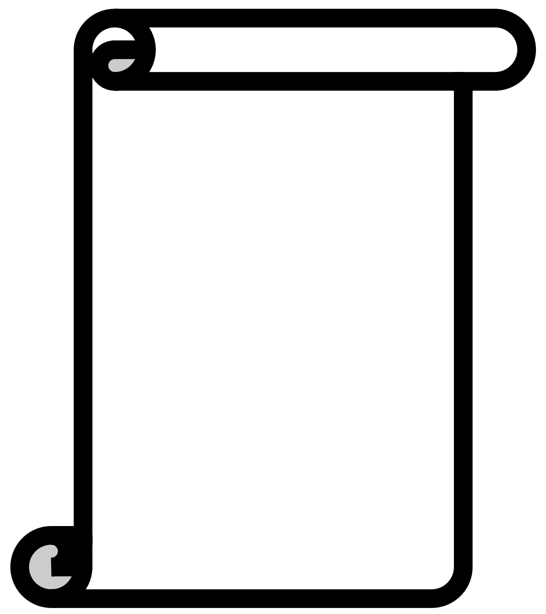

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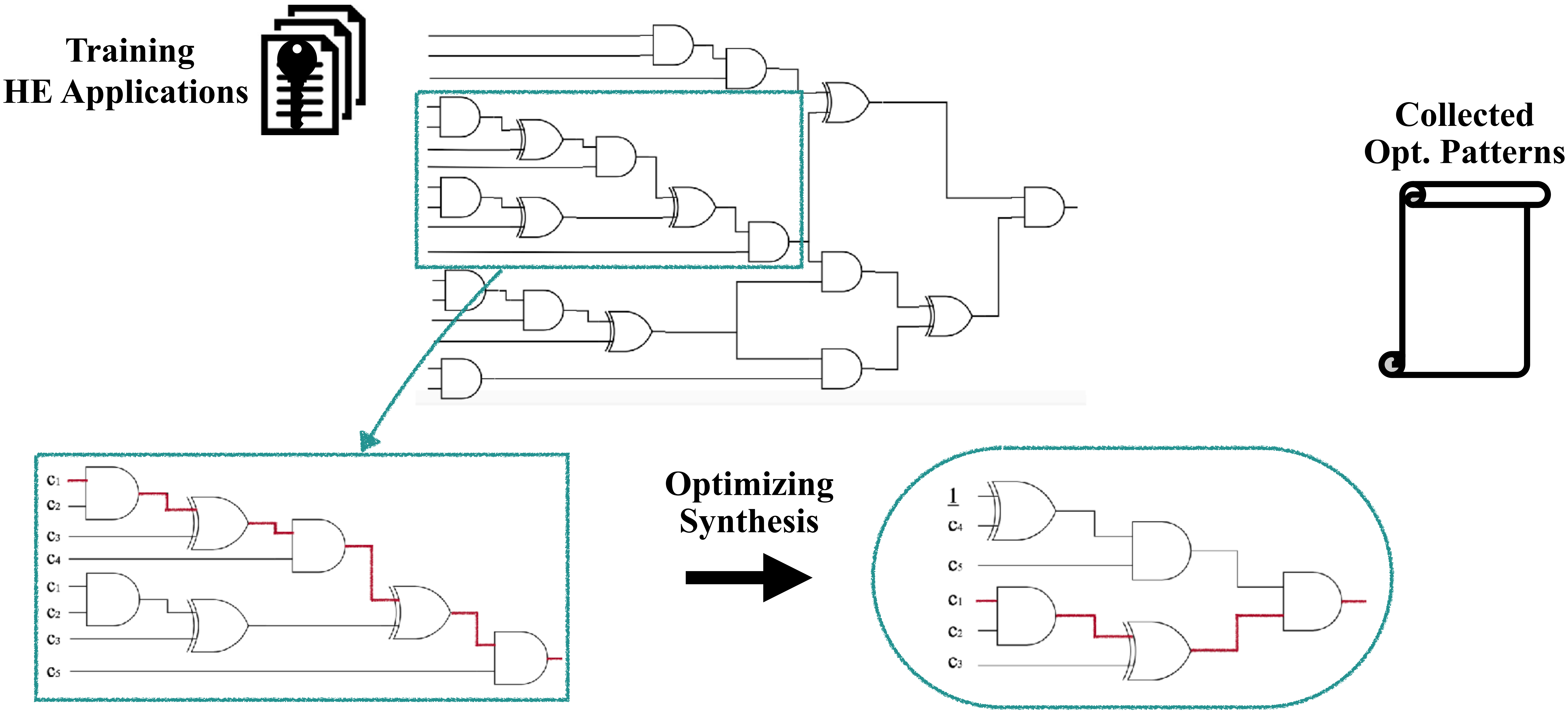

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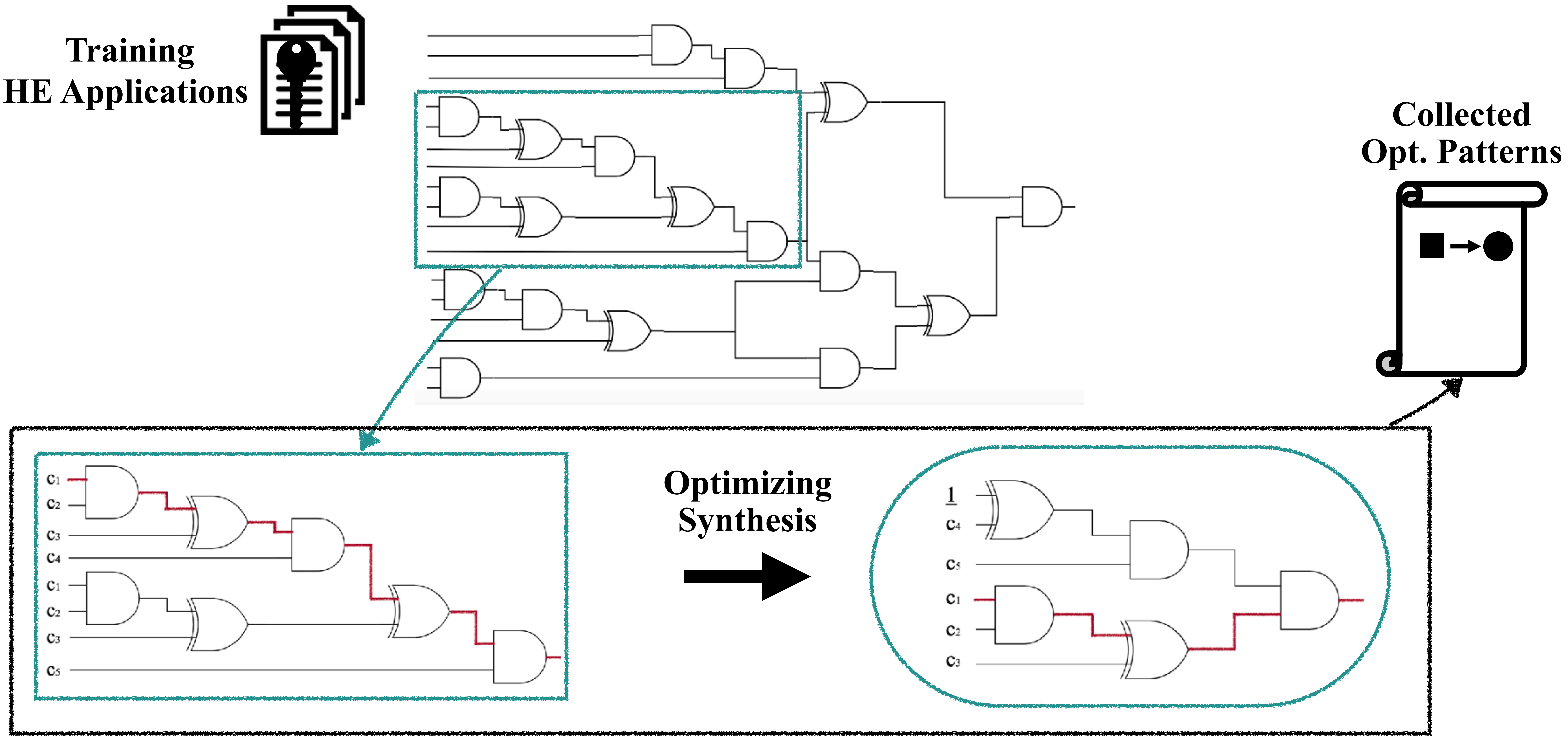

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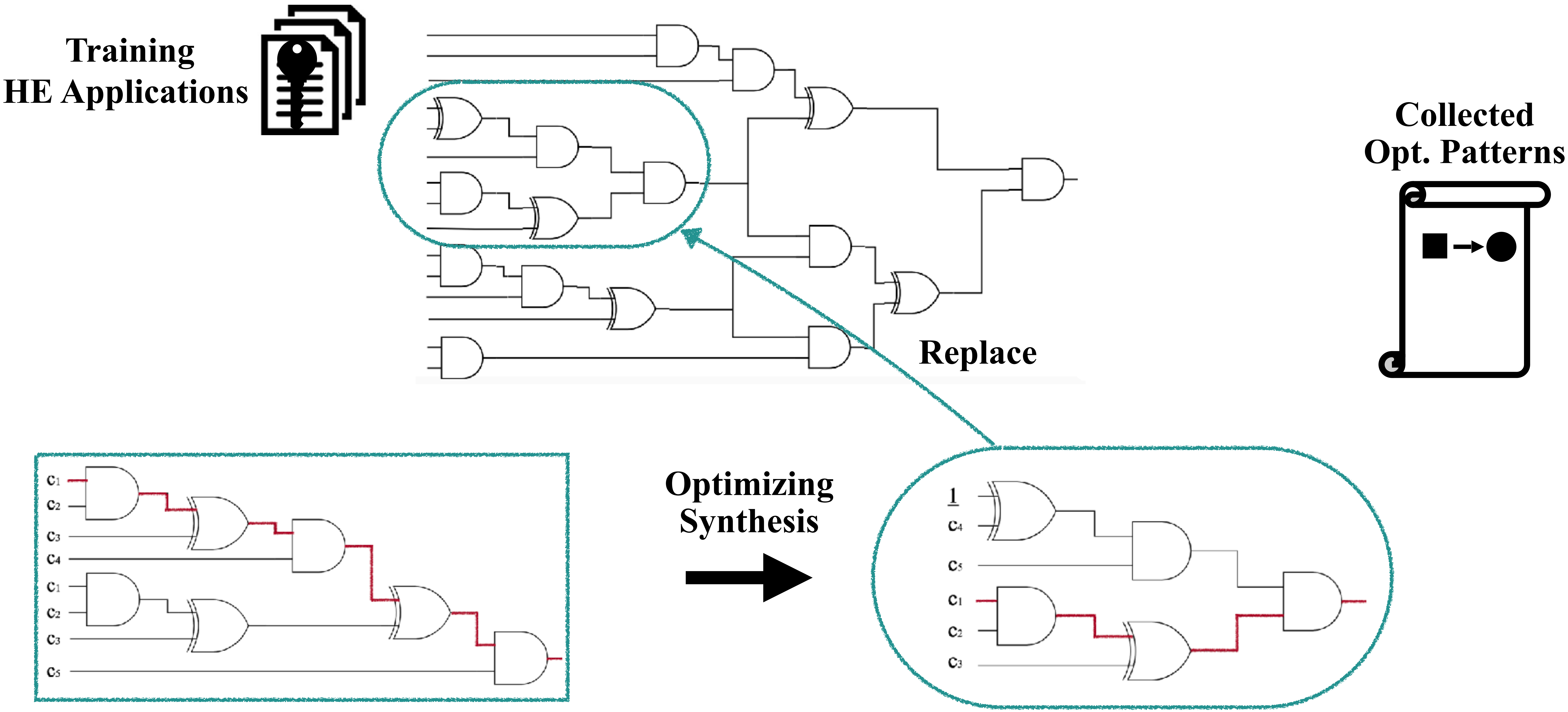

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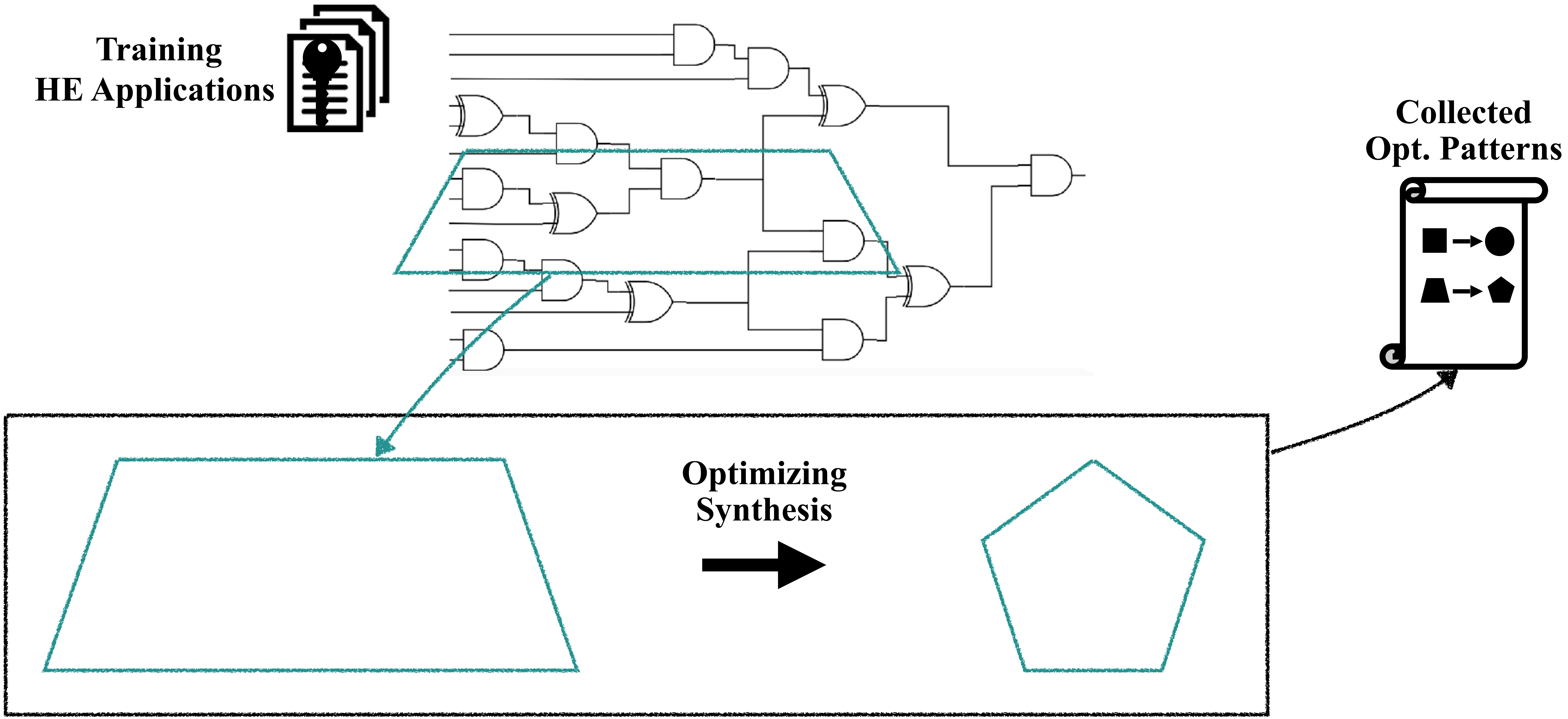

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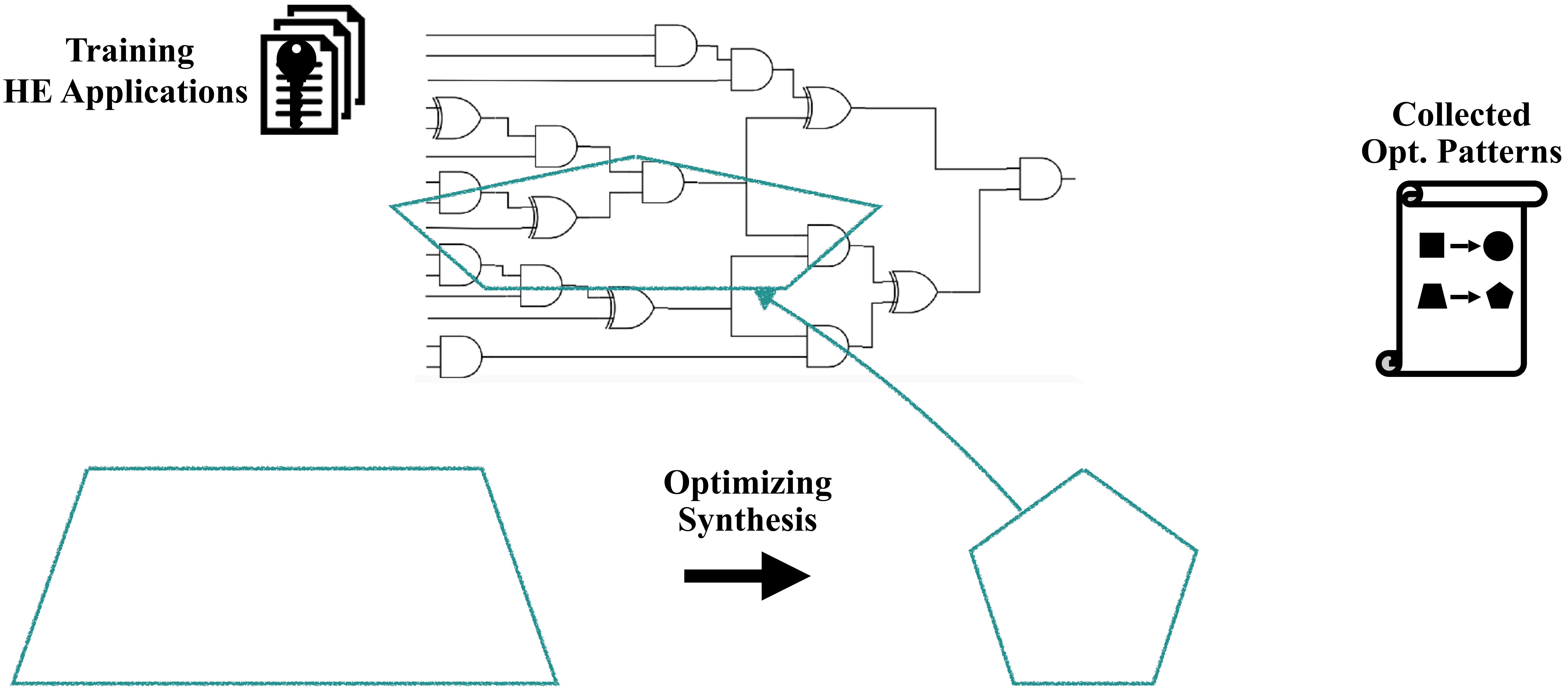

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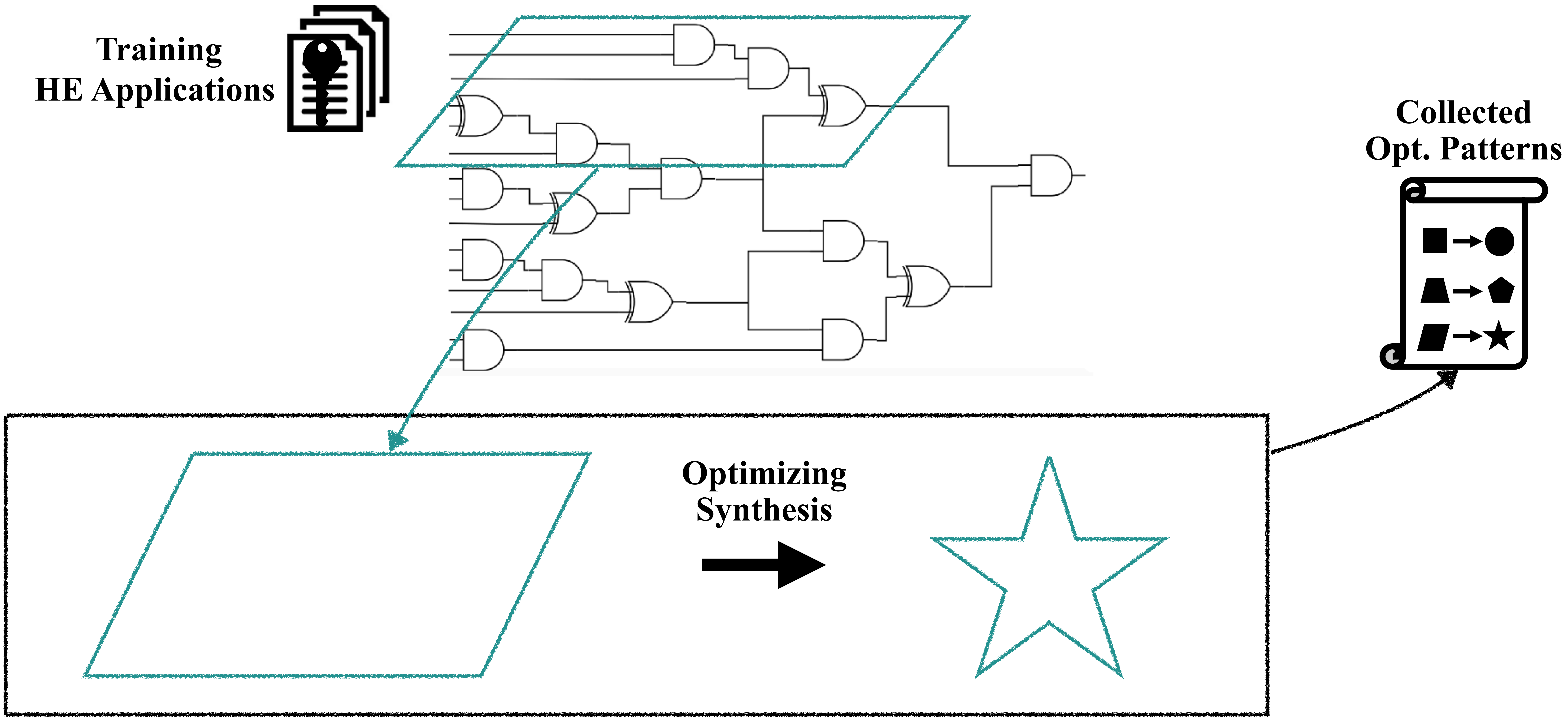

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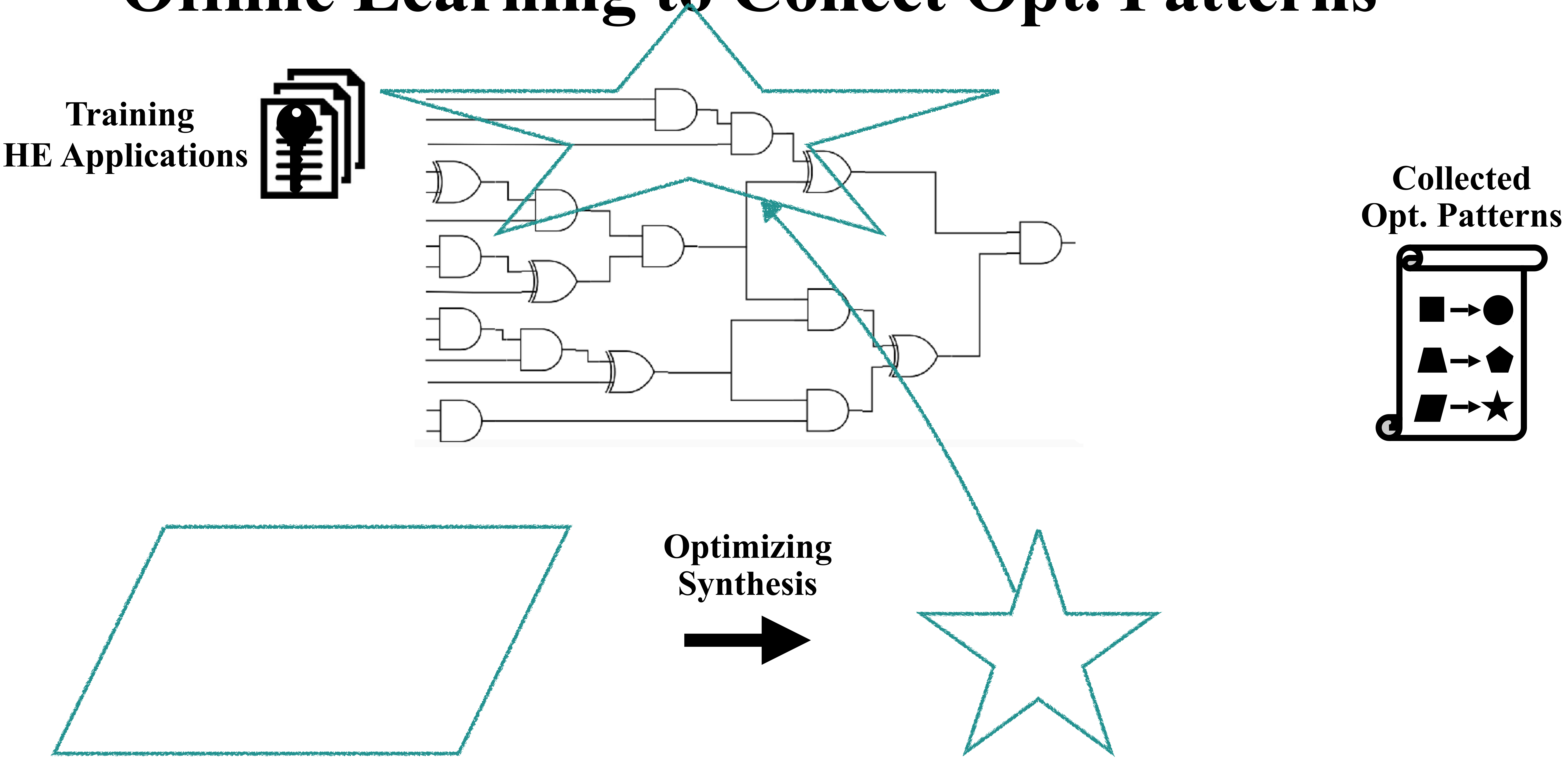
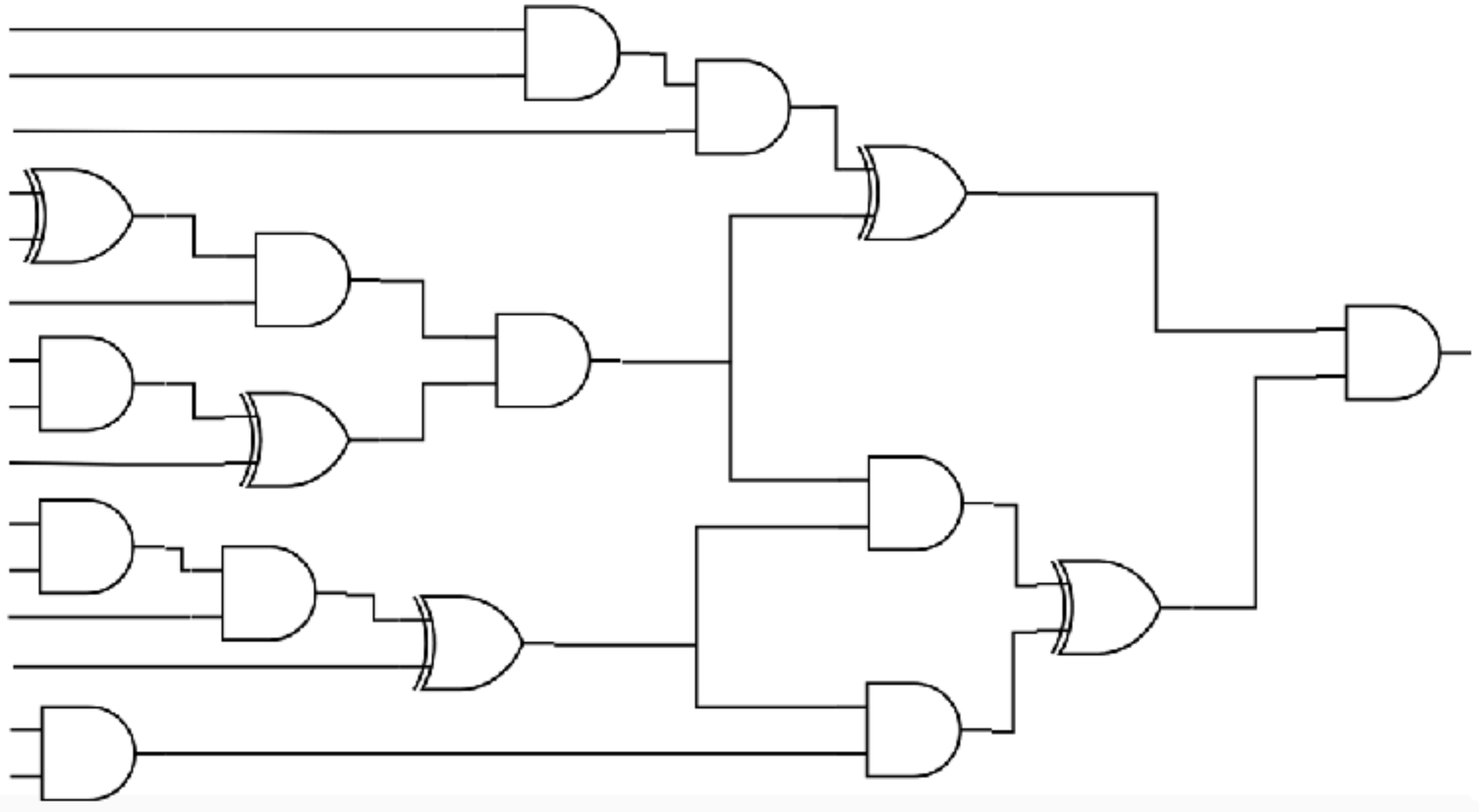
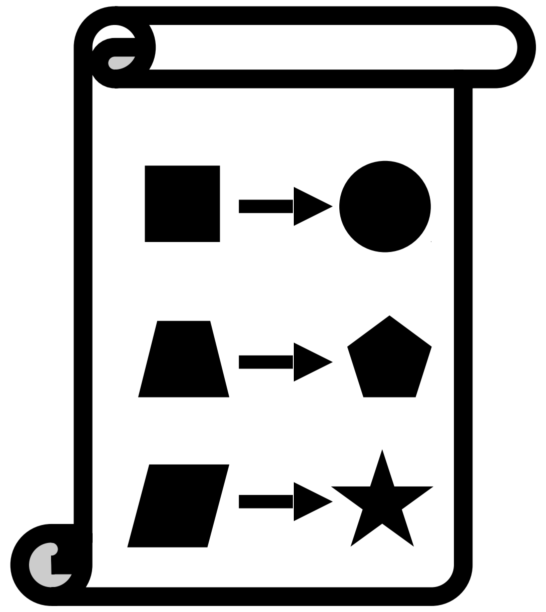


Offline Learning to Collect Opt. Patterns

Training
HE Applications

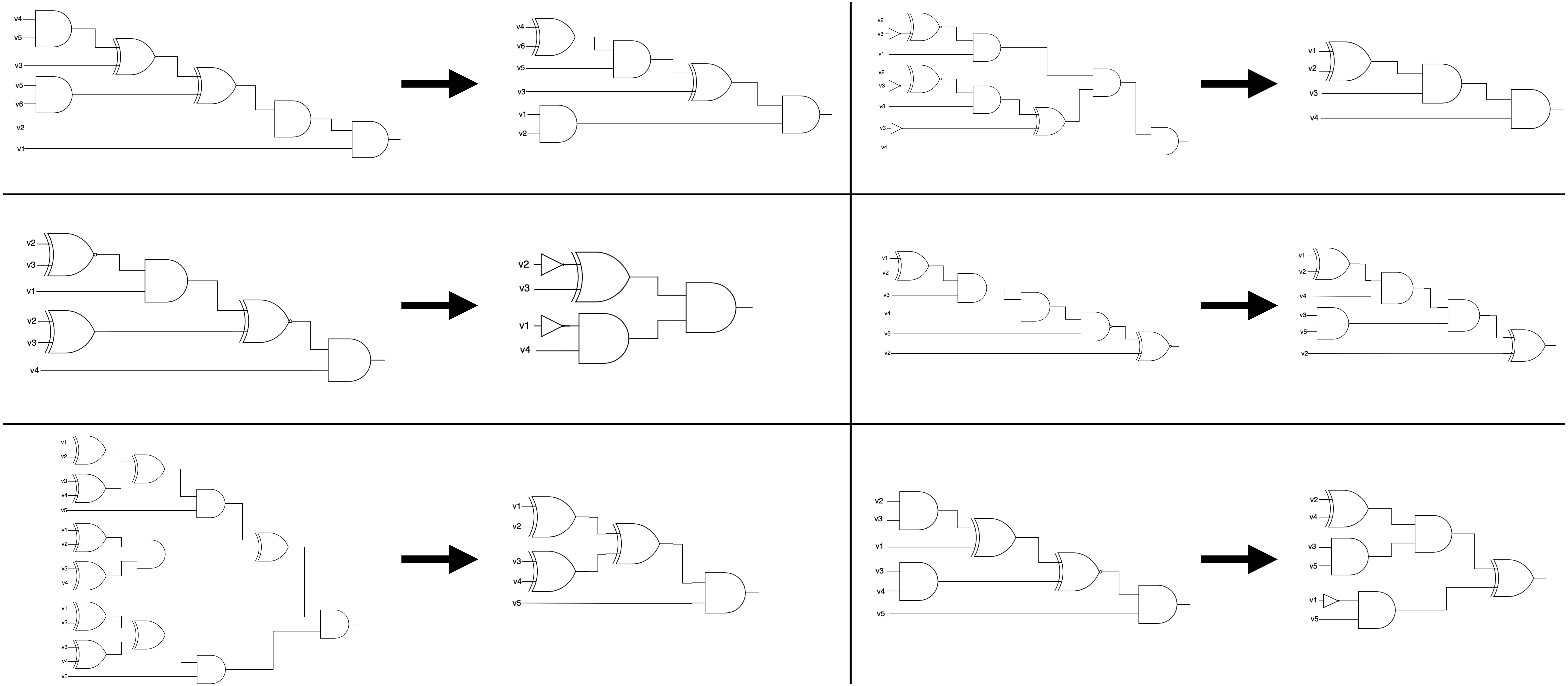




Collected
Opt. Patterns

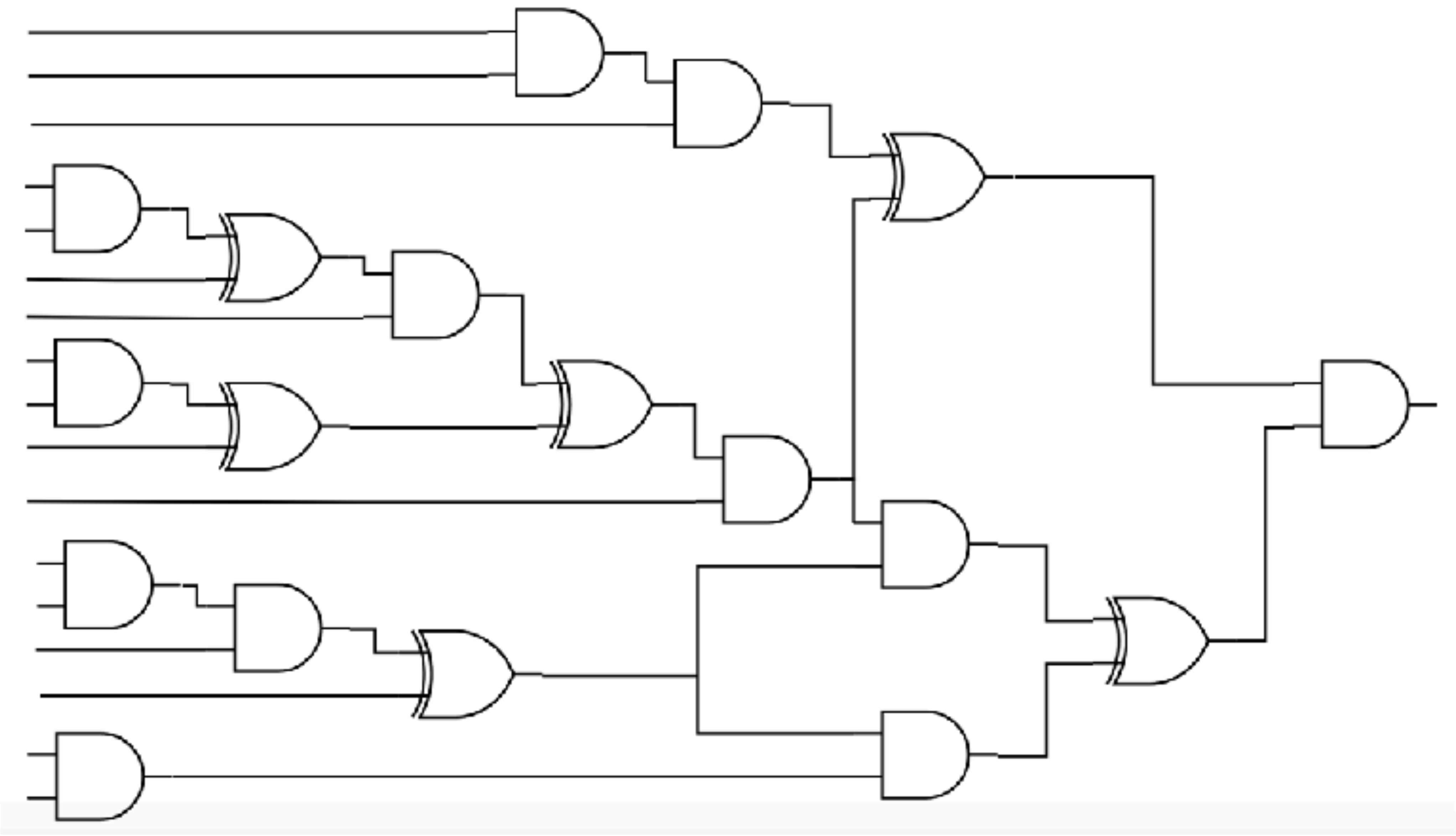
186 Opt. patterns

Learned Optimization Patterns : examples

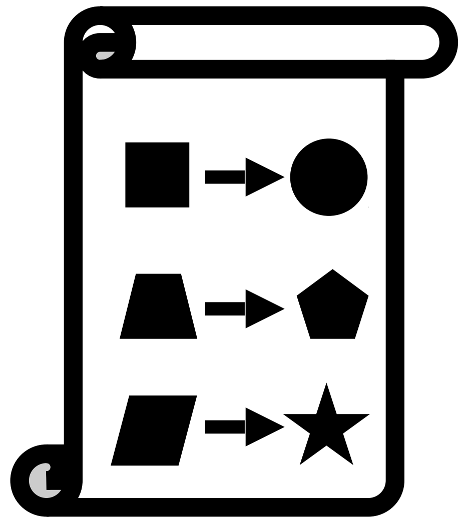


Online Rule-based Optimization

Input
HE application



Learned
Opt. Patterns



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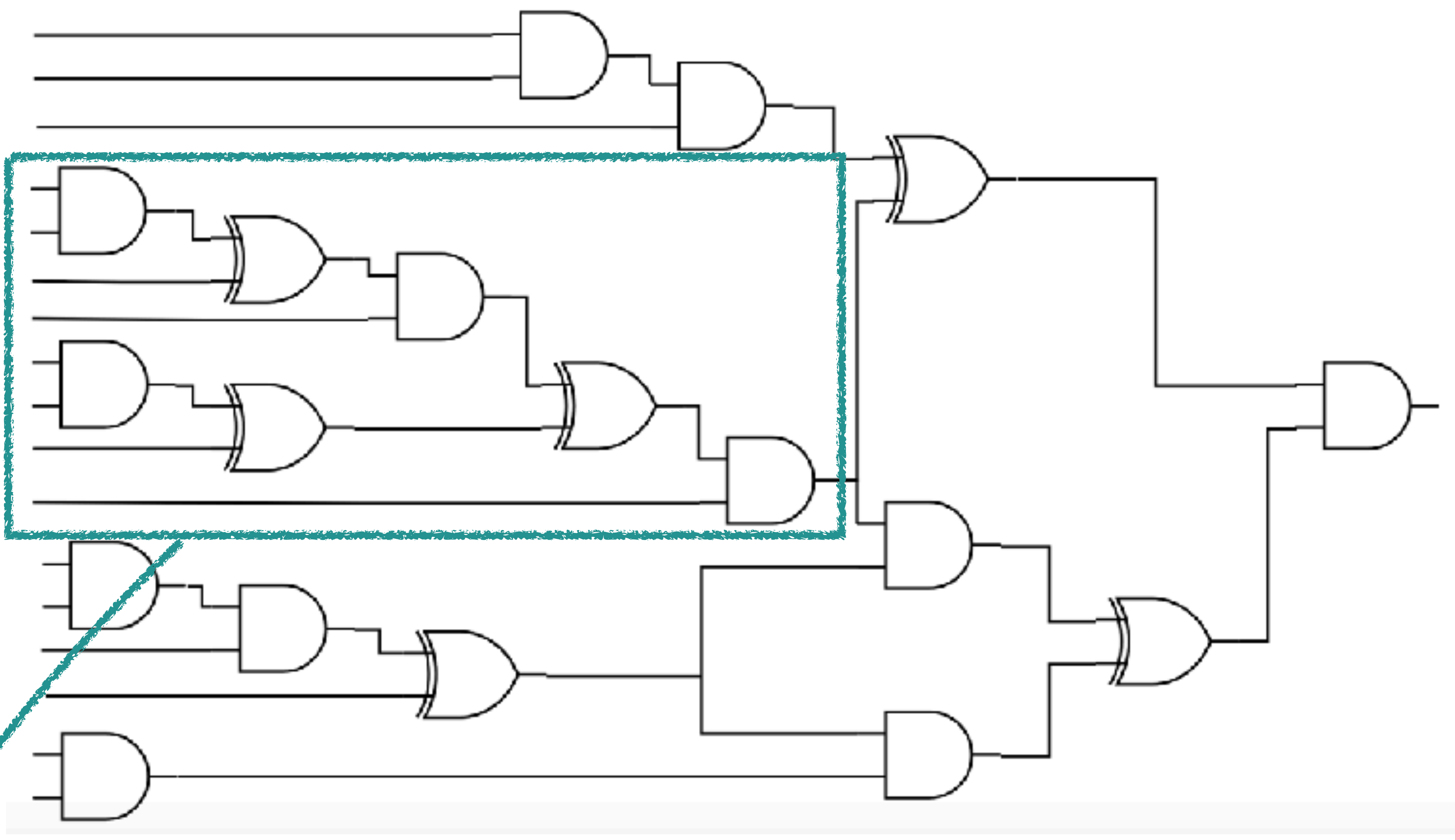
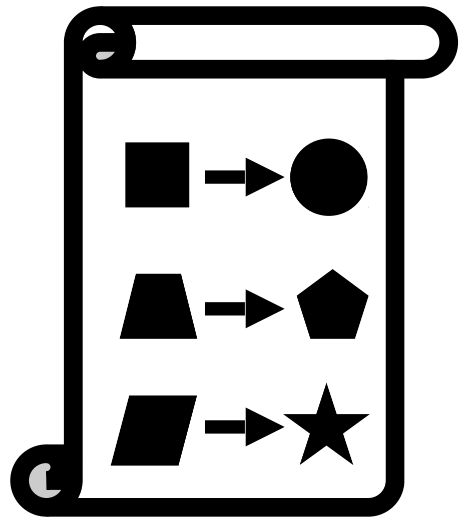
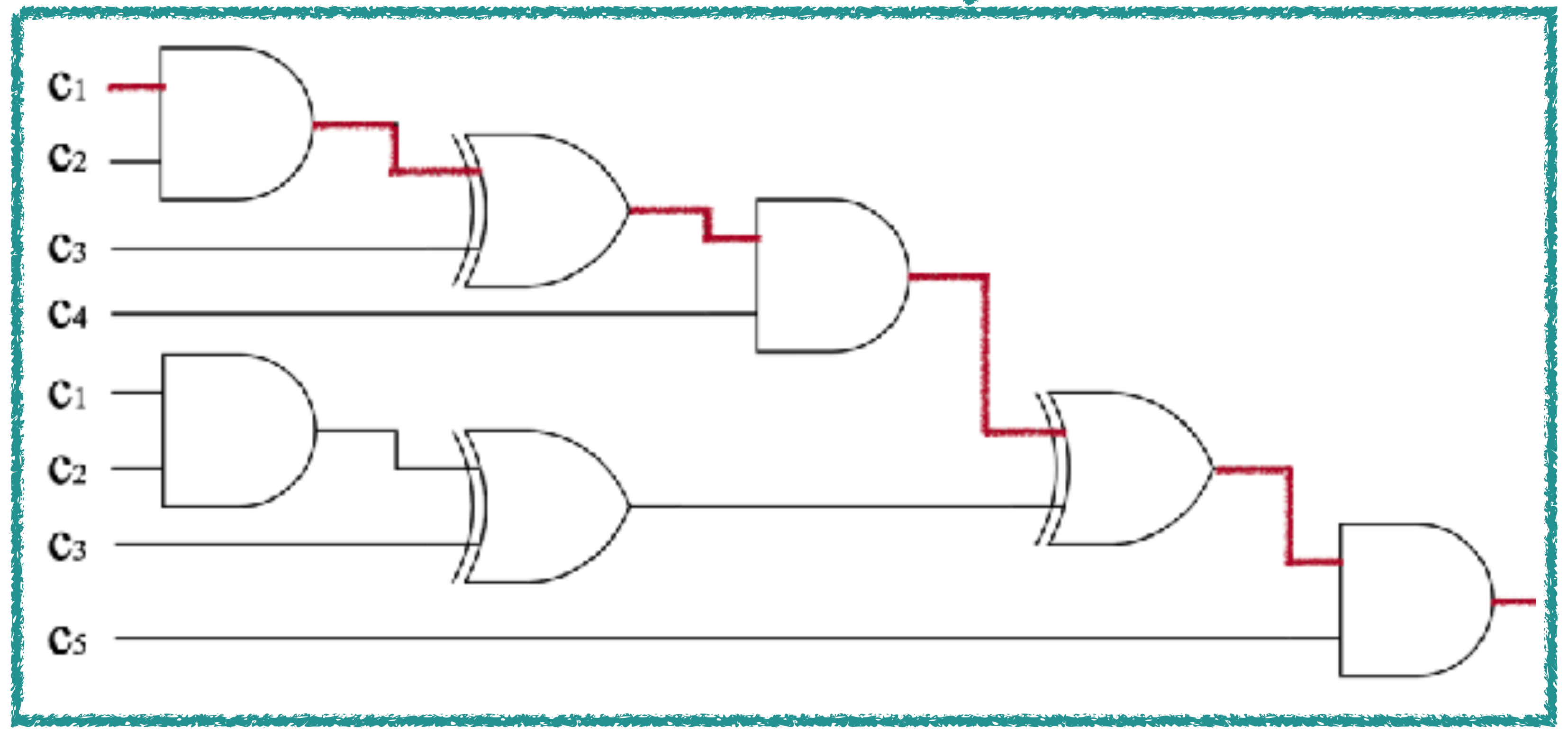
Input
HE application

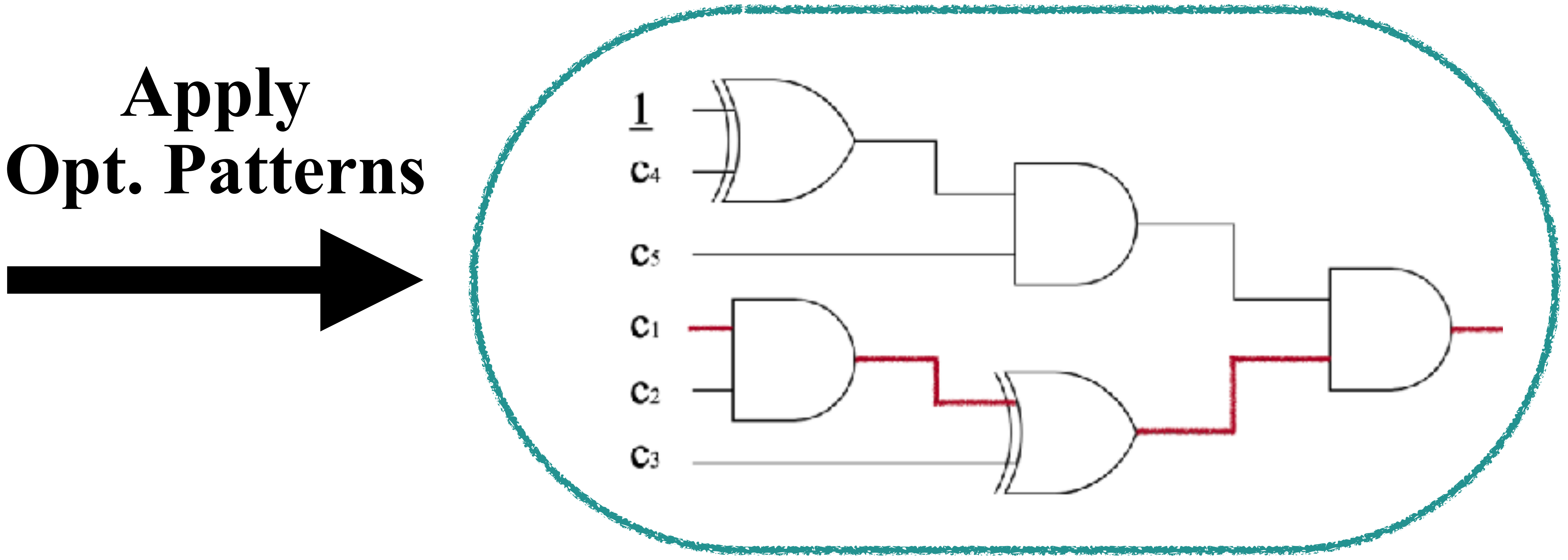


Learned
Opt. Patterns

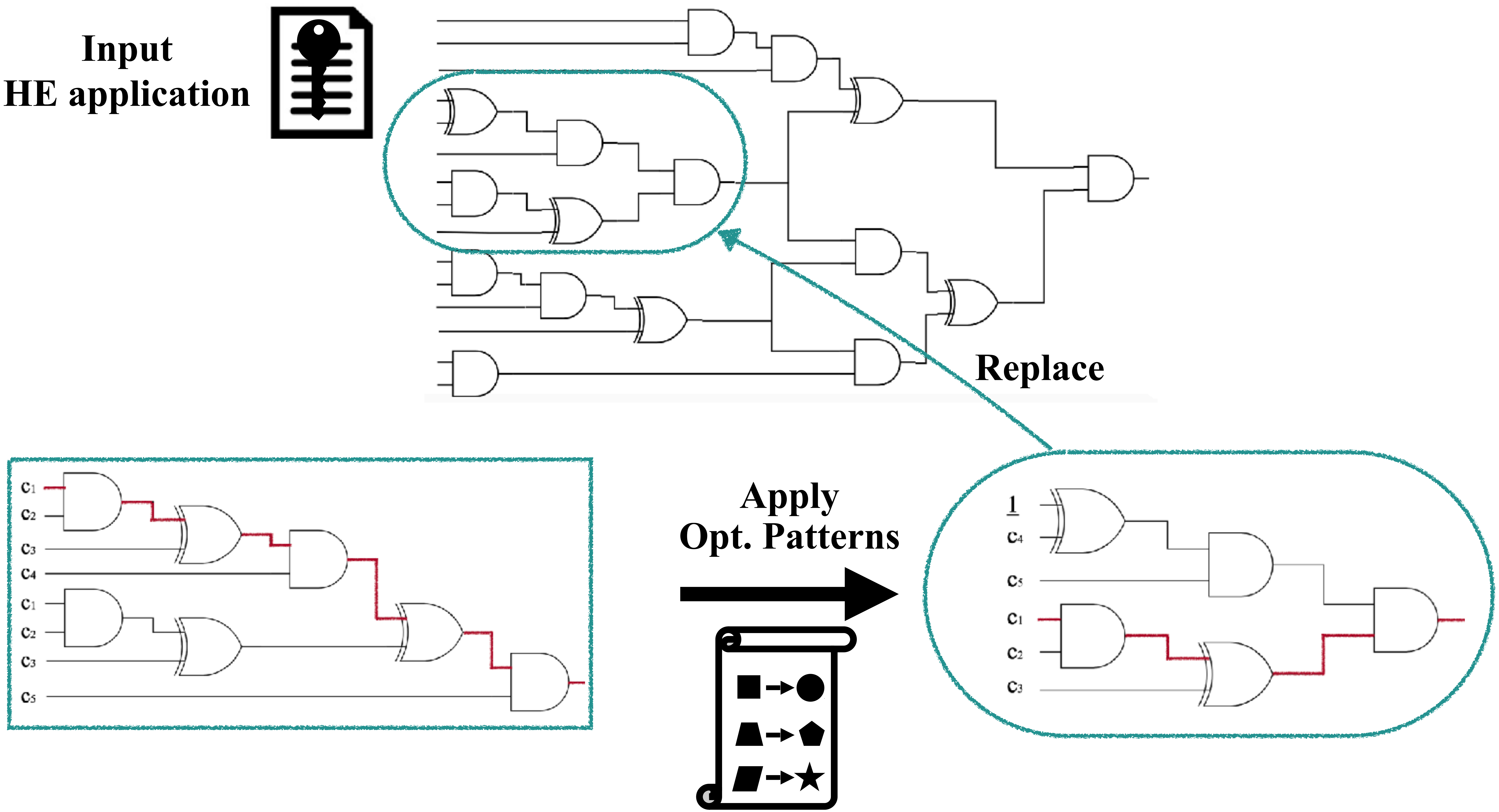




Apply
Opt. Patterns

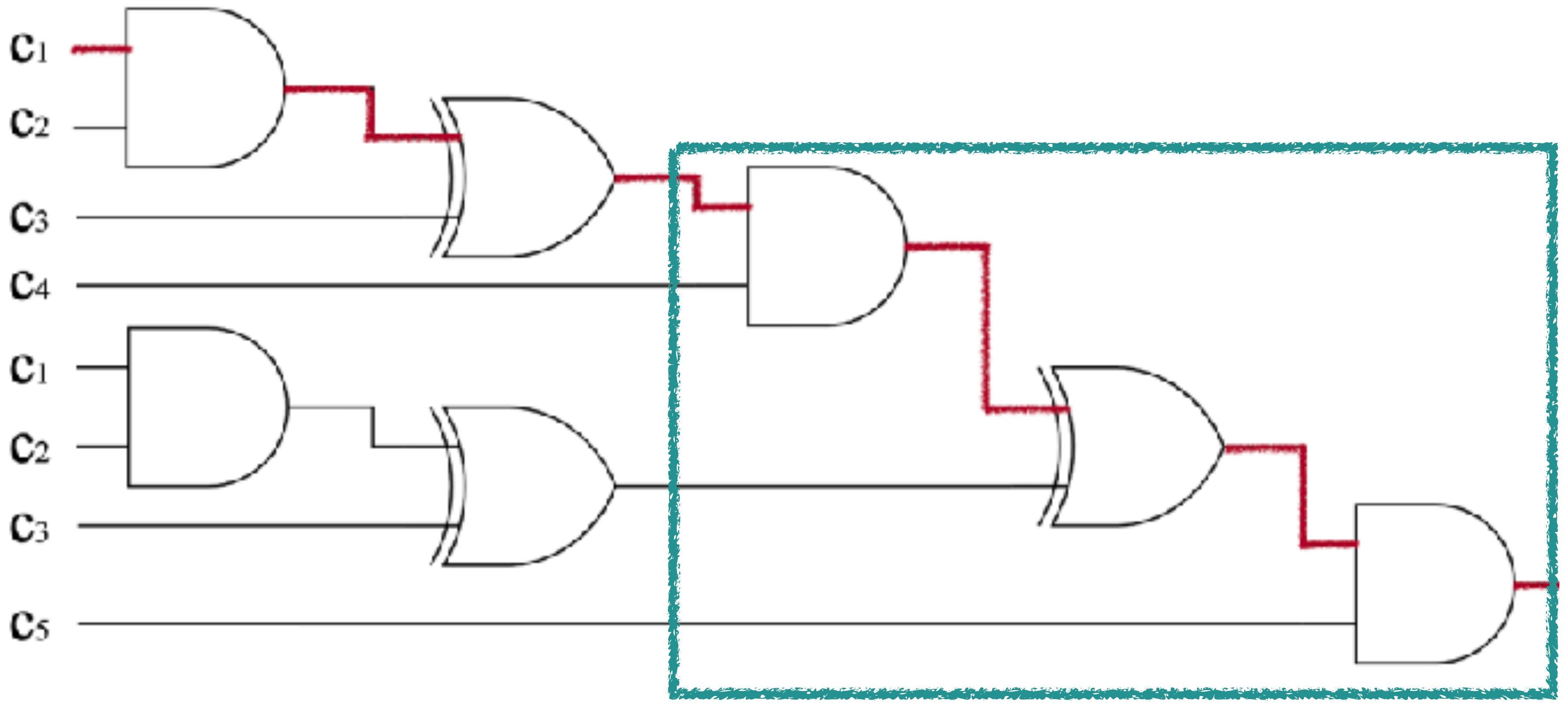


Online Rule-based Optimization

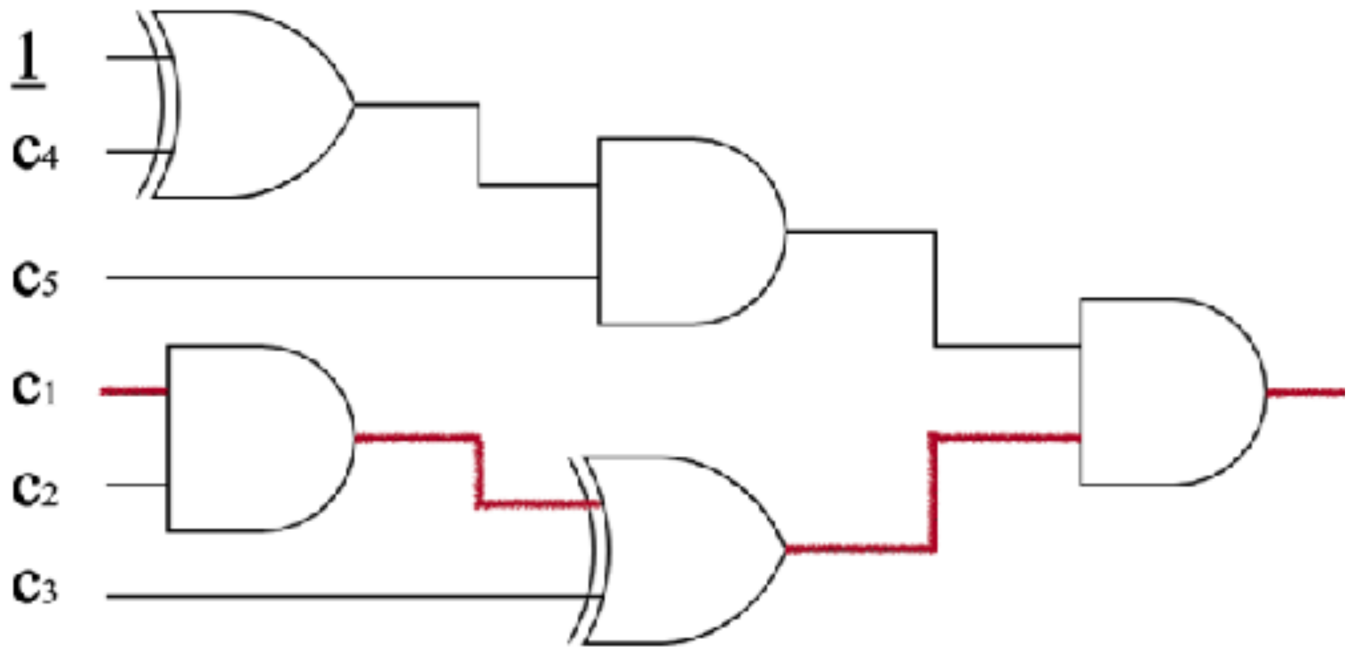


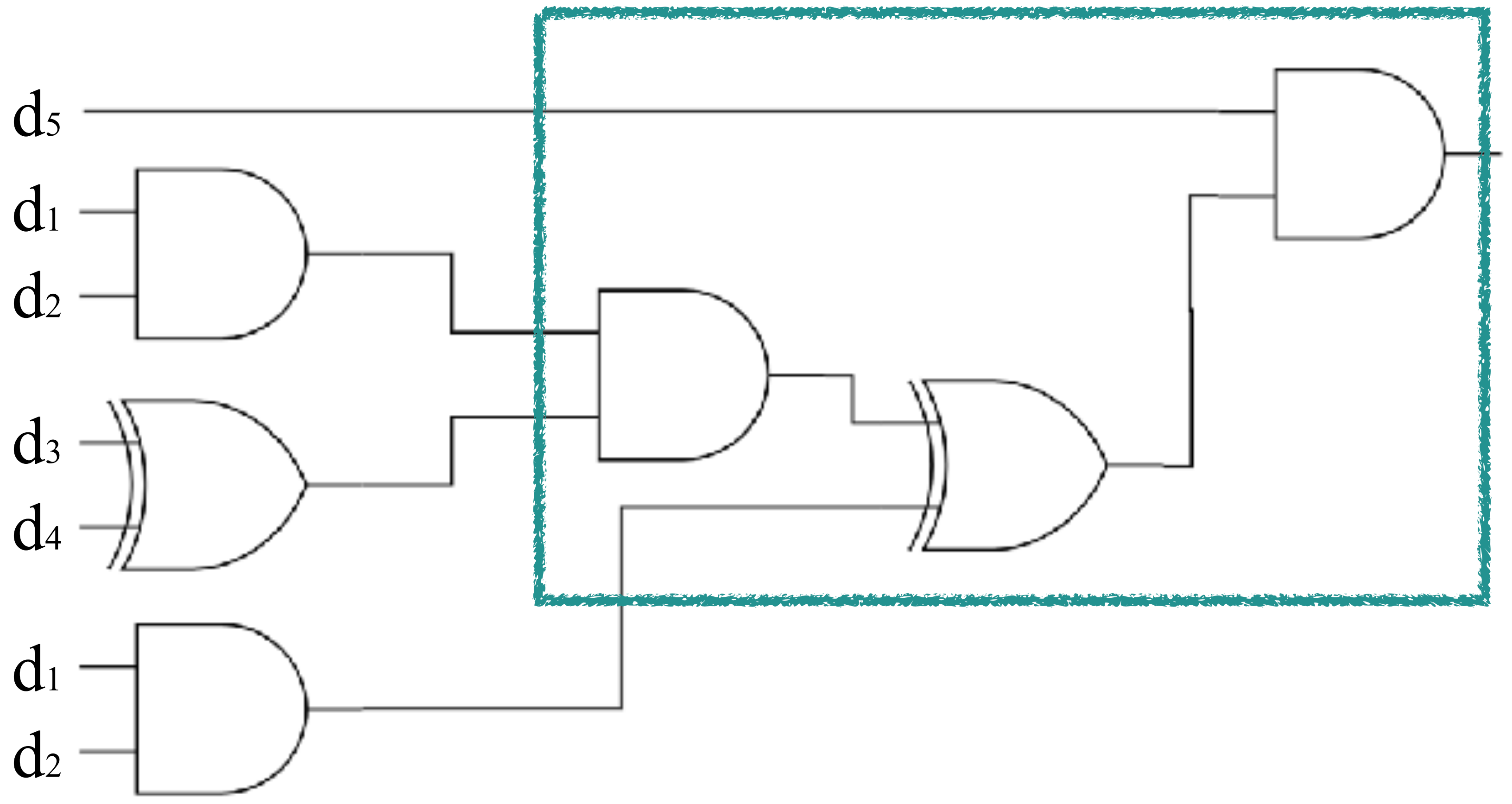
Applying Learned Optimization Patterns (1/2)

Syntactic Matching is Not Effective



Learned Opt. Patterns



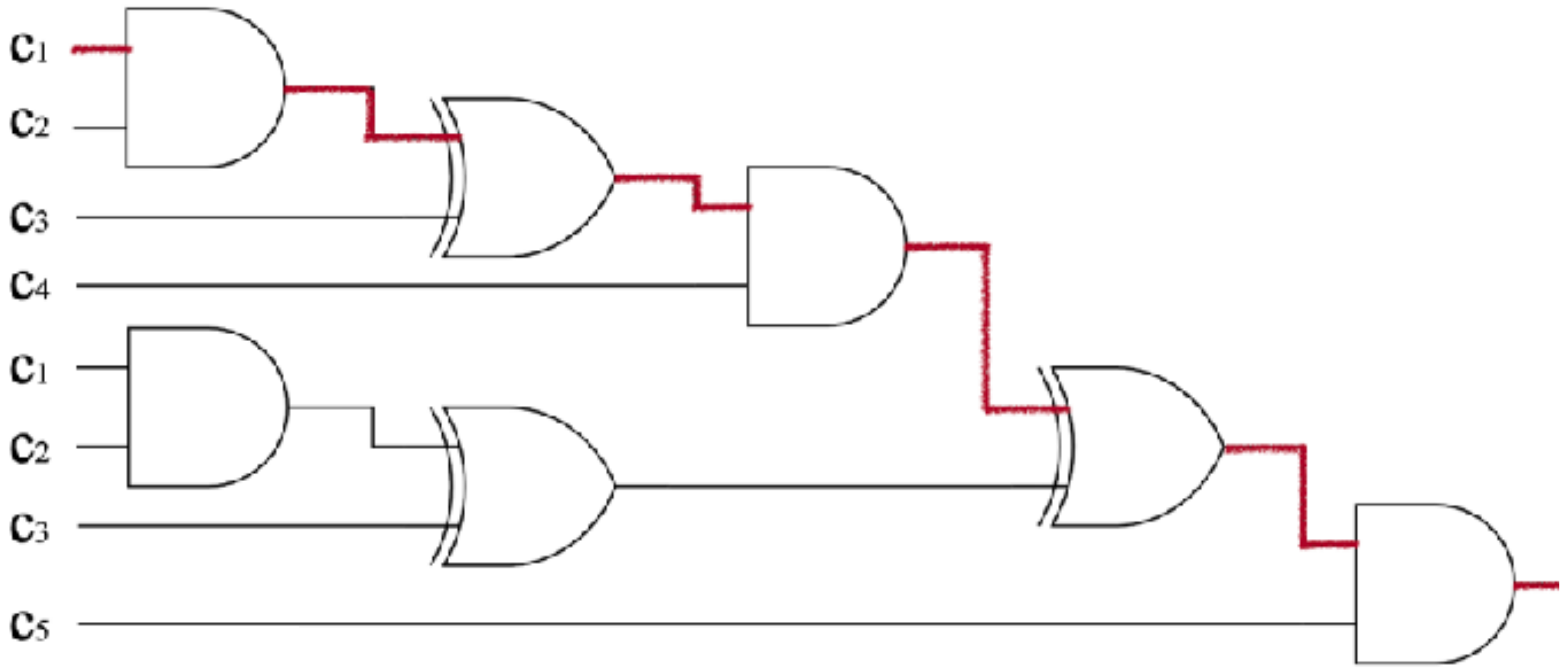


New Input Circuit Optimization

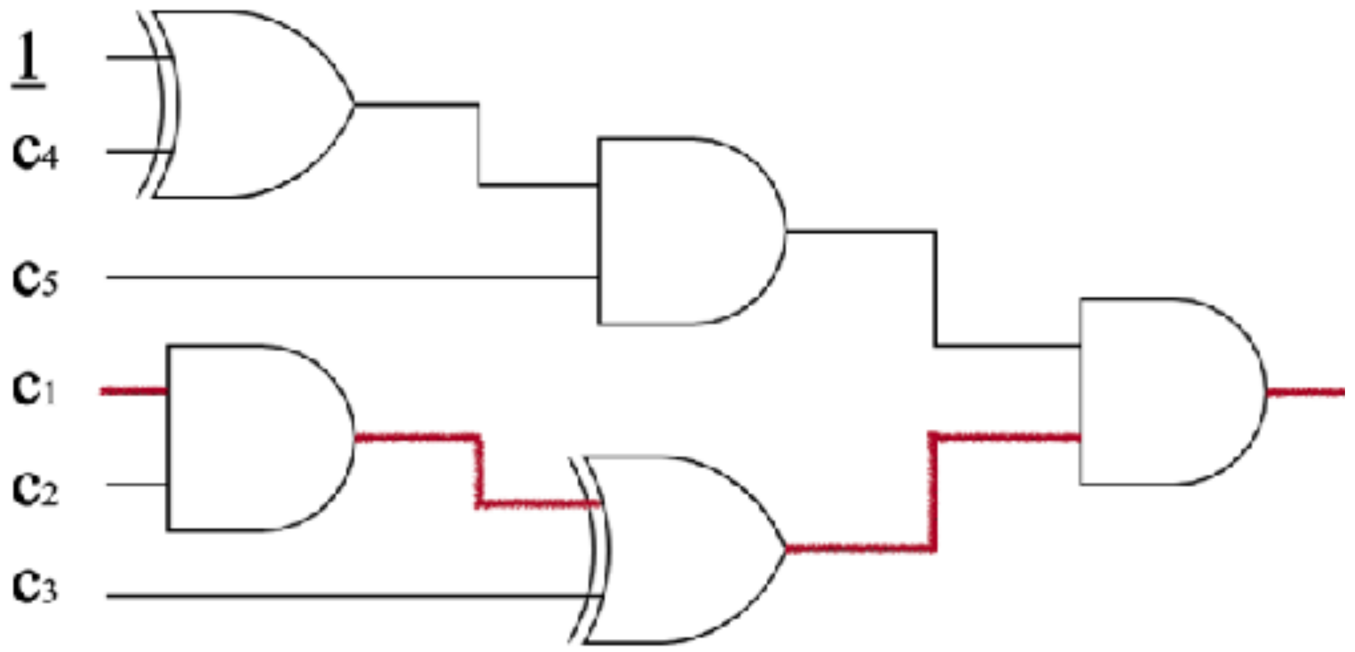
?

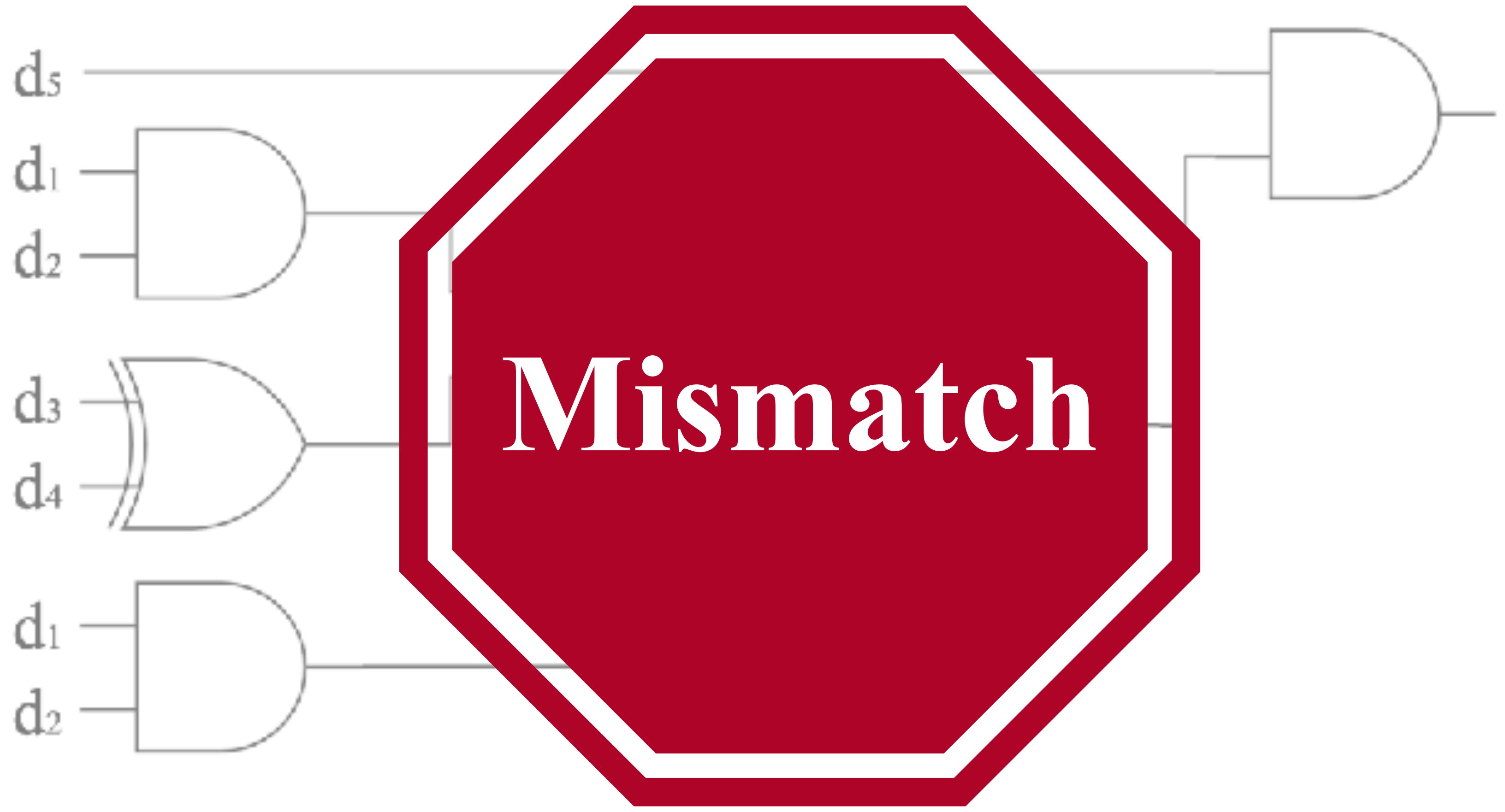
Applying Learned Optimization Patterns (1/2)

Syntactic Matching is Not Effective



Learned
Opt. Patterns



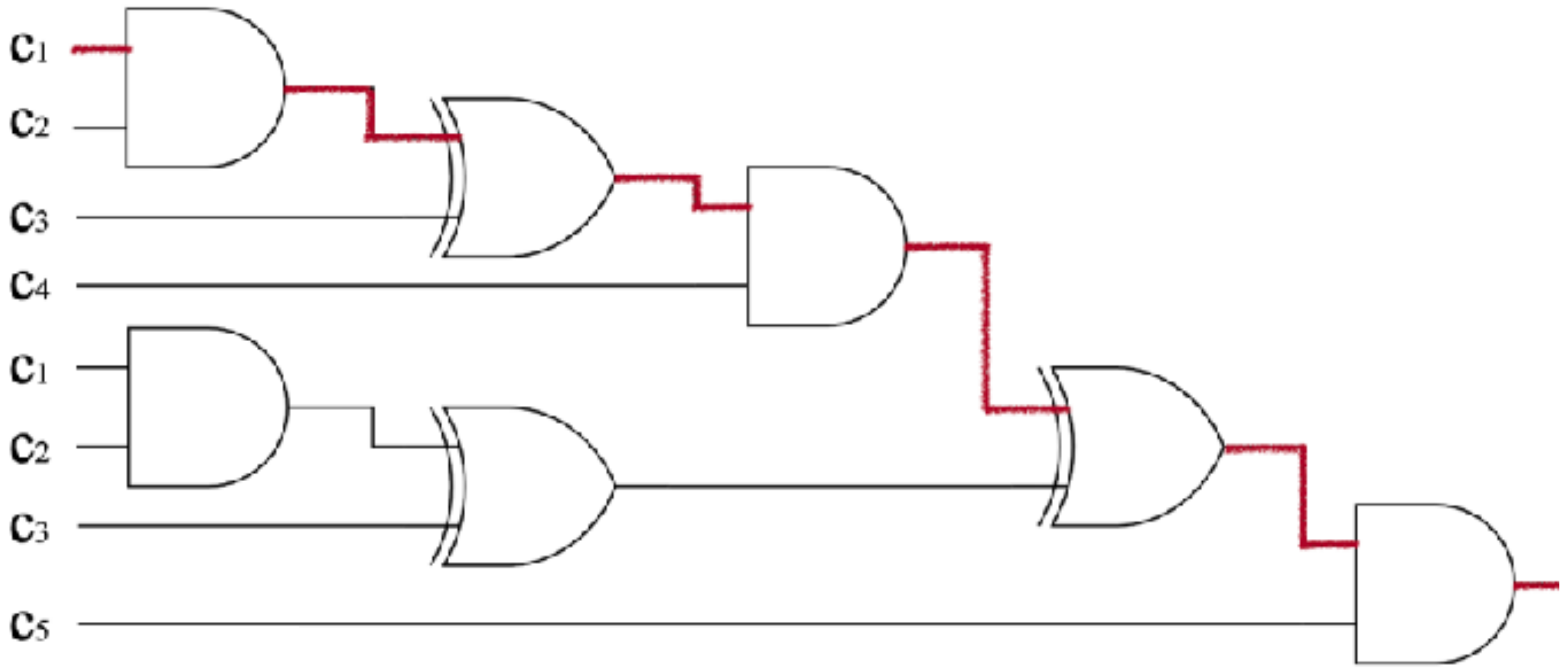


New Input Circuit
Optimization

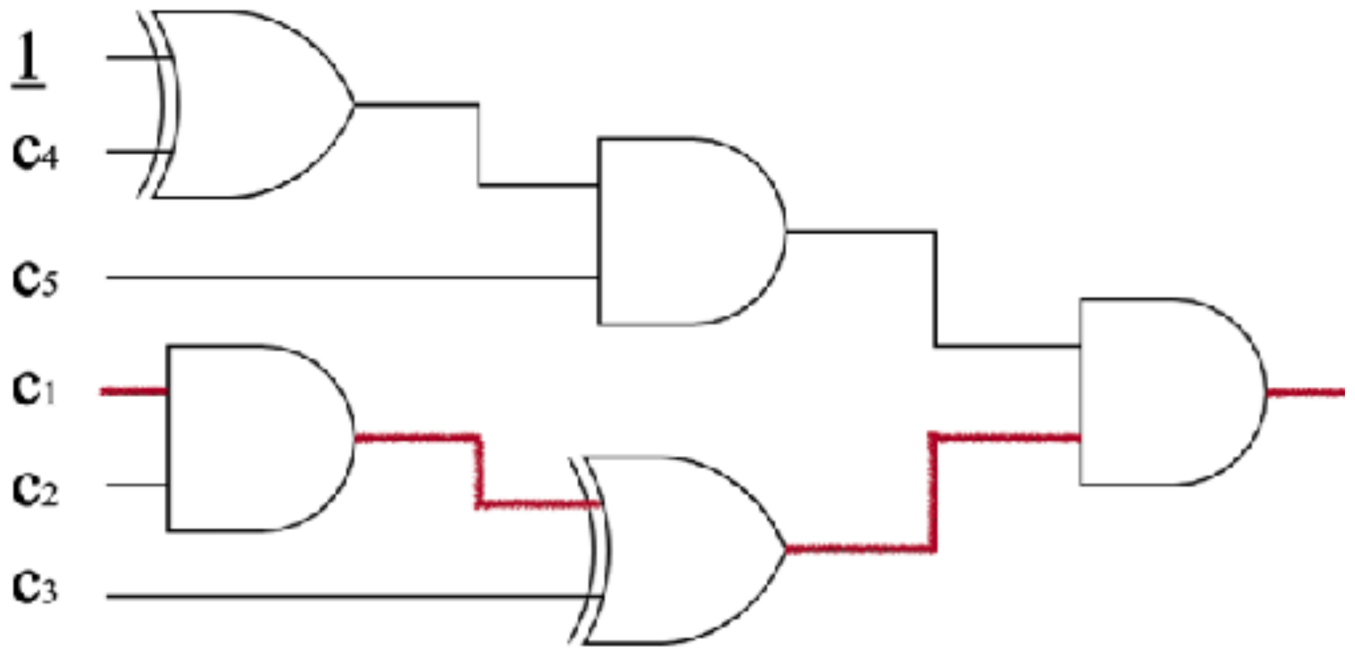
?

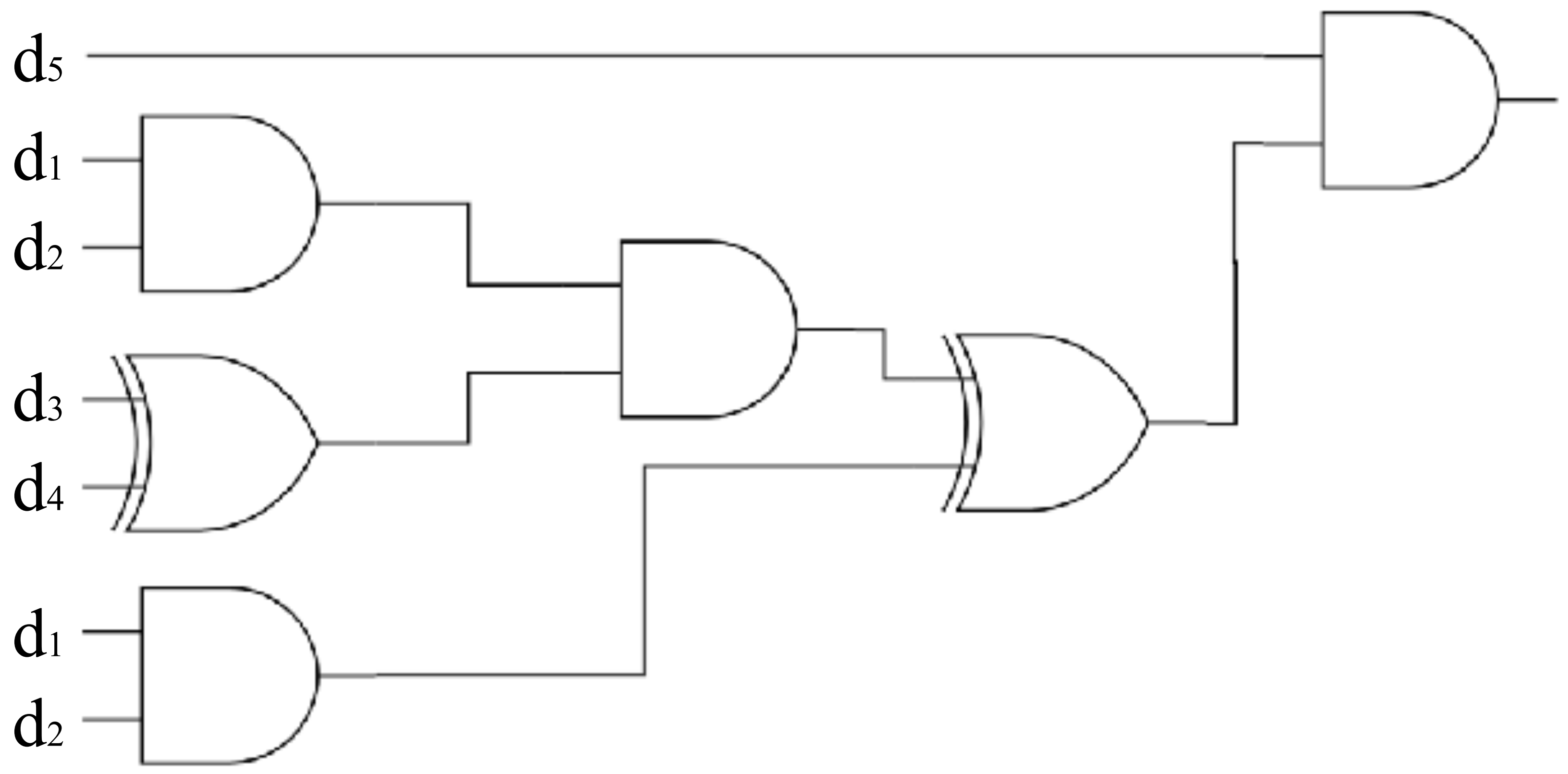
Applying Learned Optimization Patterns (2/2)

Normalization + Equational Matching



Learned Opt. Patterns



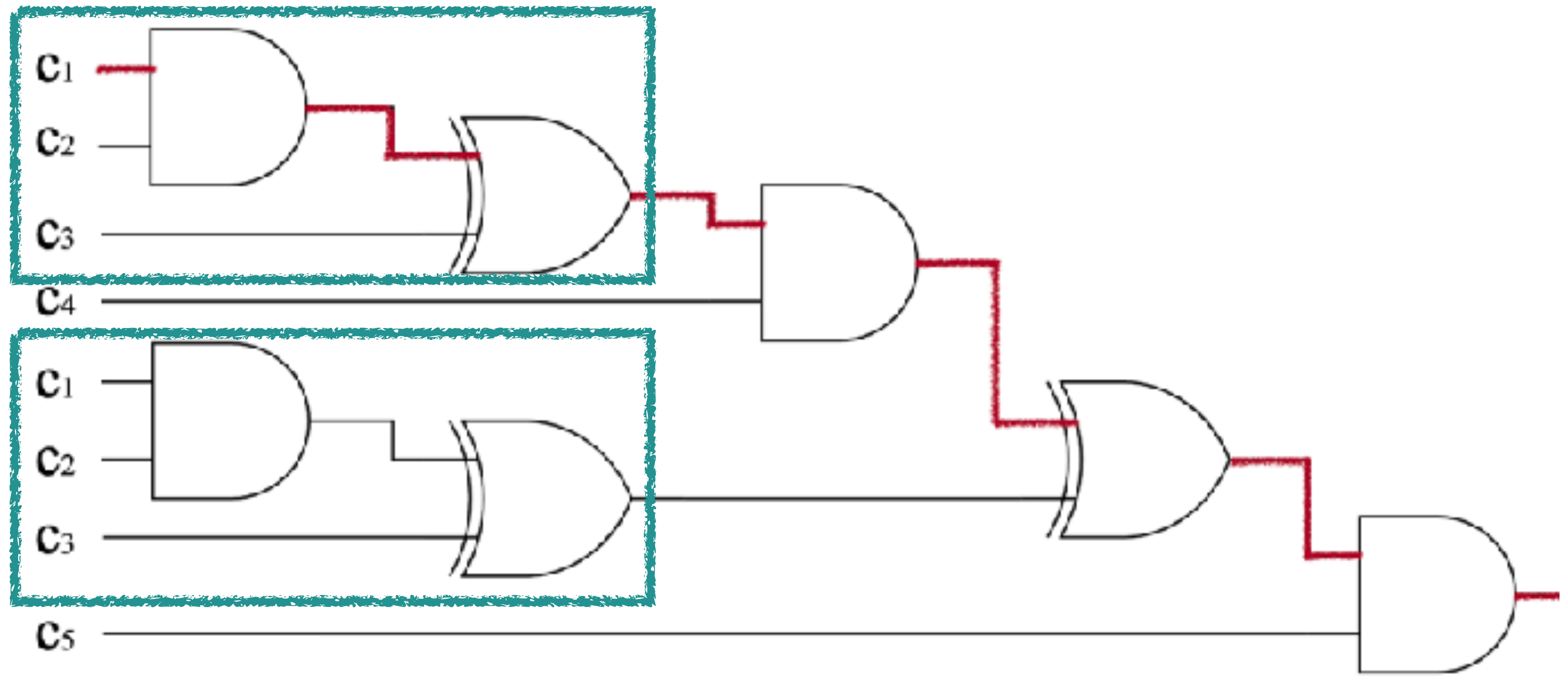


New Input Circuit Optimization

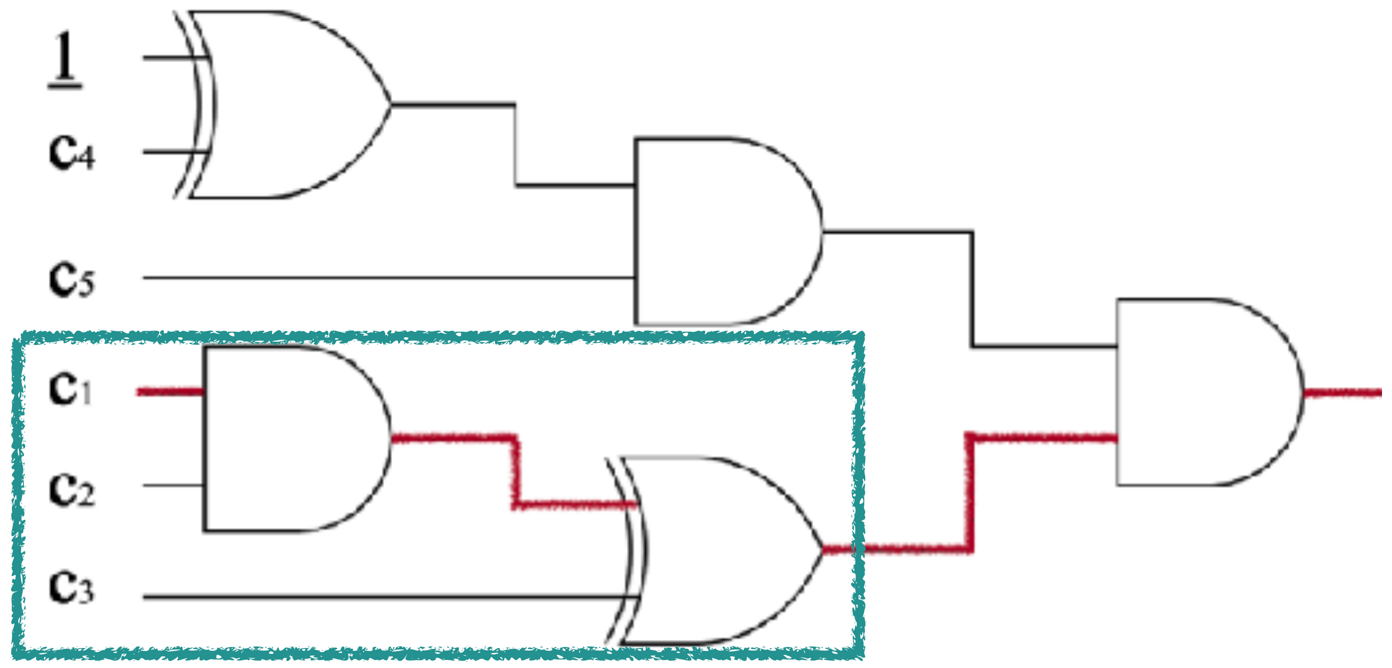
?

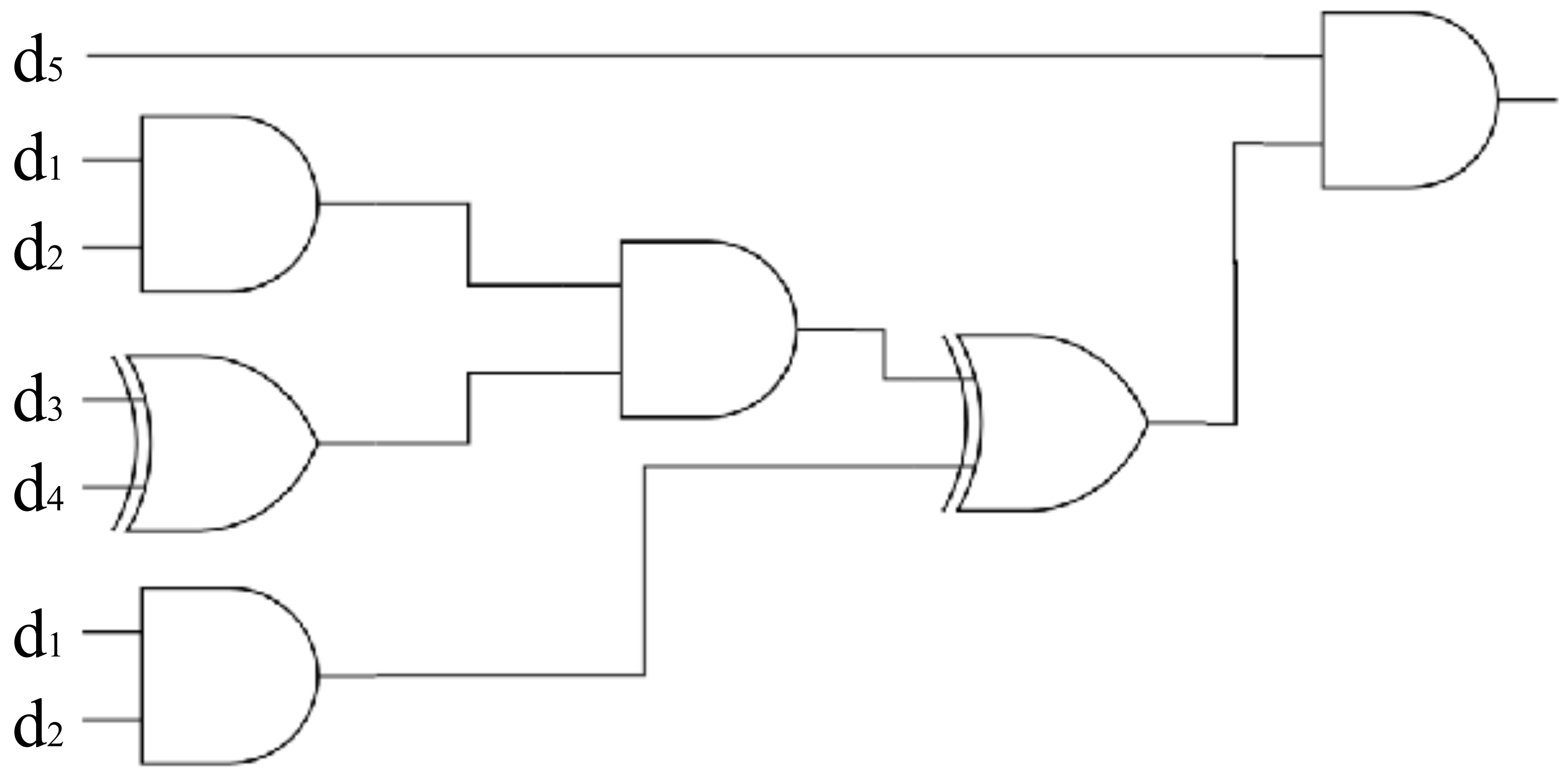
Applying Learned Optimization Patterns (2/2)

Normalization + Equational Matching



Learned Opt. Patterns



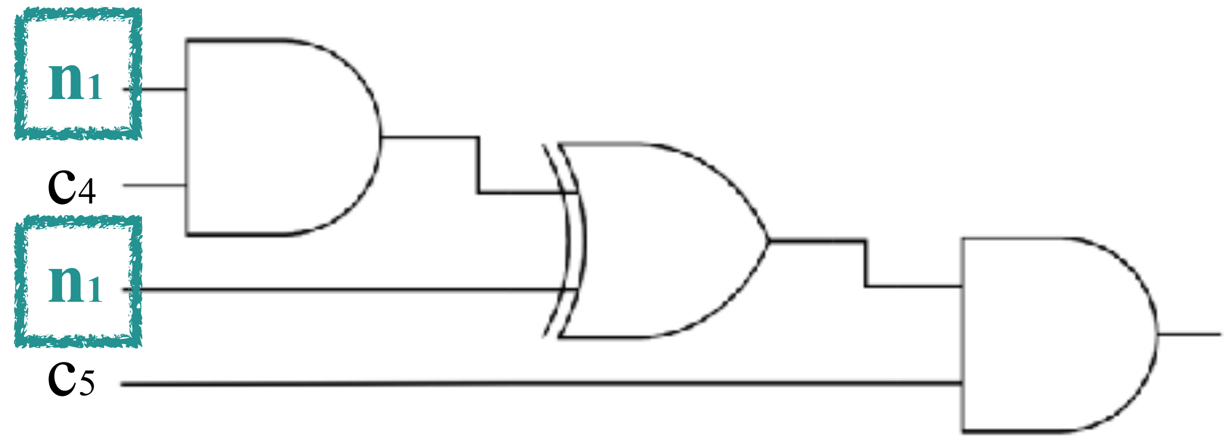


New Input Circuit Optimization

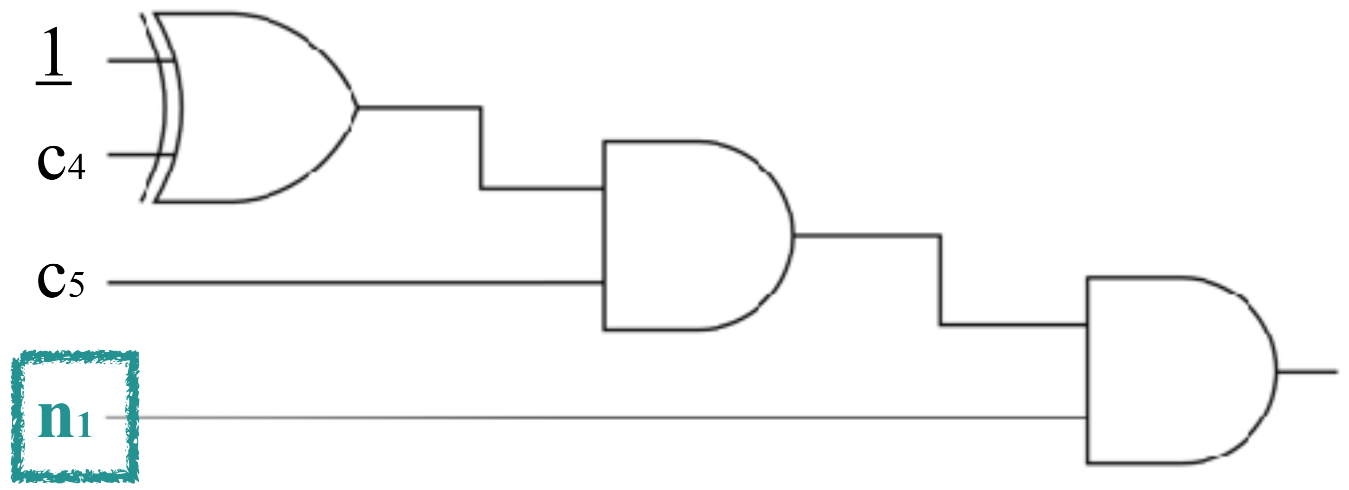
?

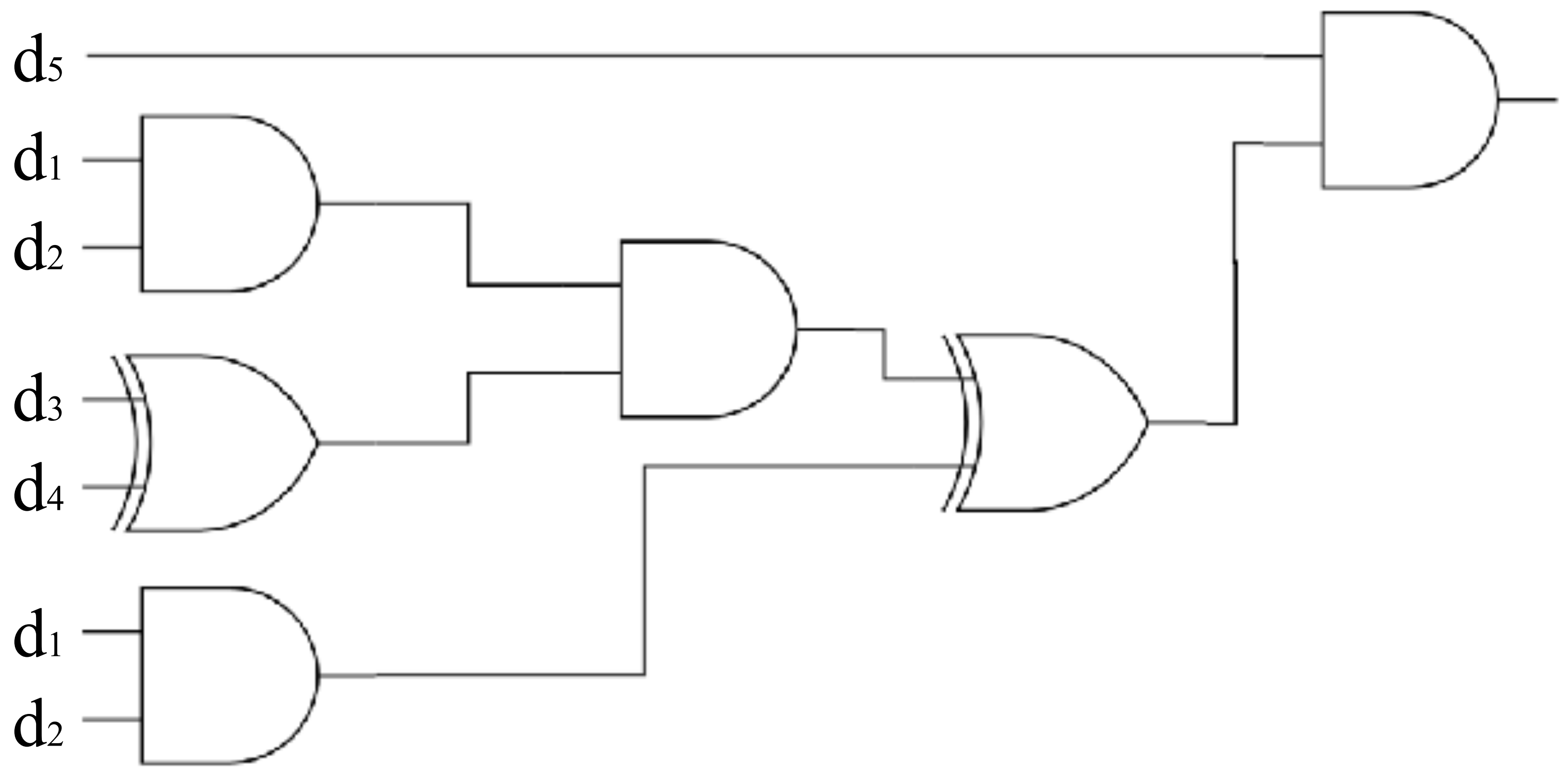
Applying Learned Optimization Patterns (2/2)

Normalization + Equational Matching



Normalized Opt. Patterns





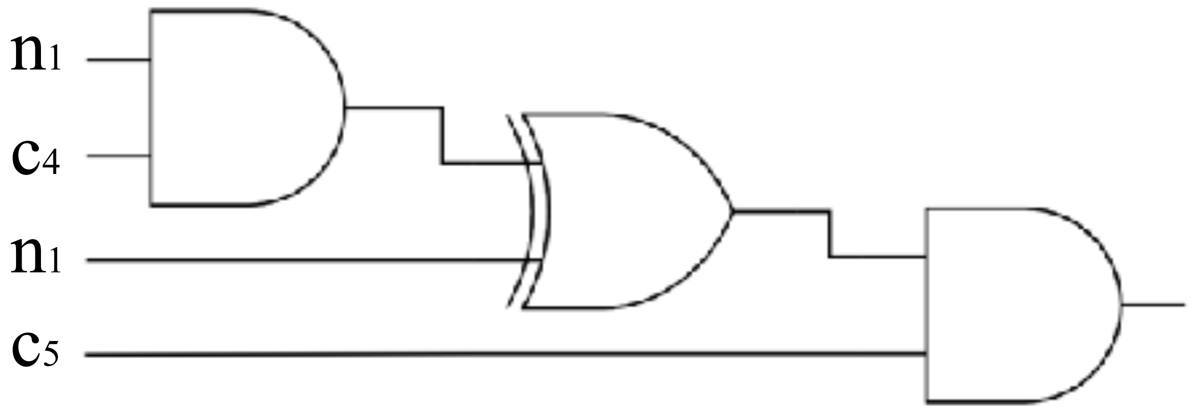
New Input Circuit Optimization

?

Applying Learned Optimization Patterns (2/2)

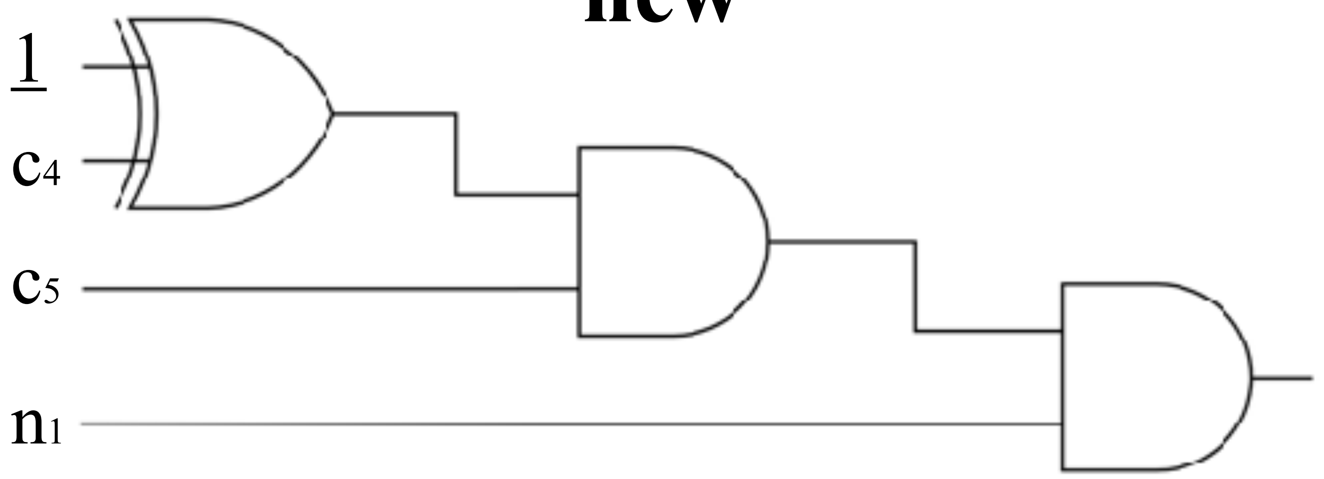
Normalization + Equational Matching

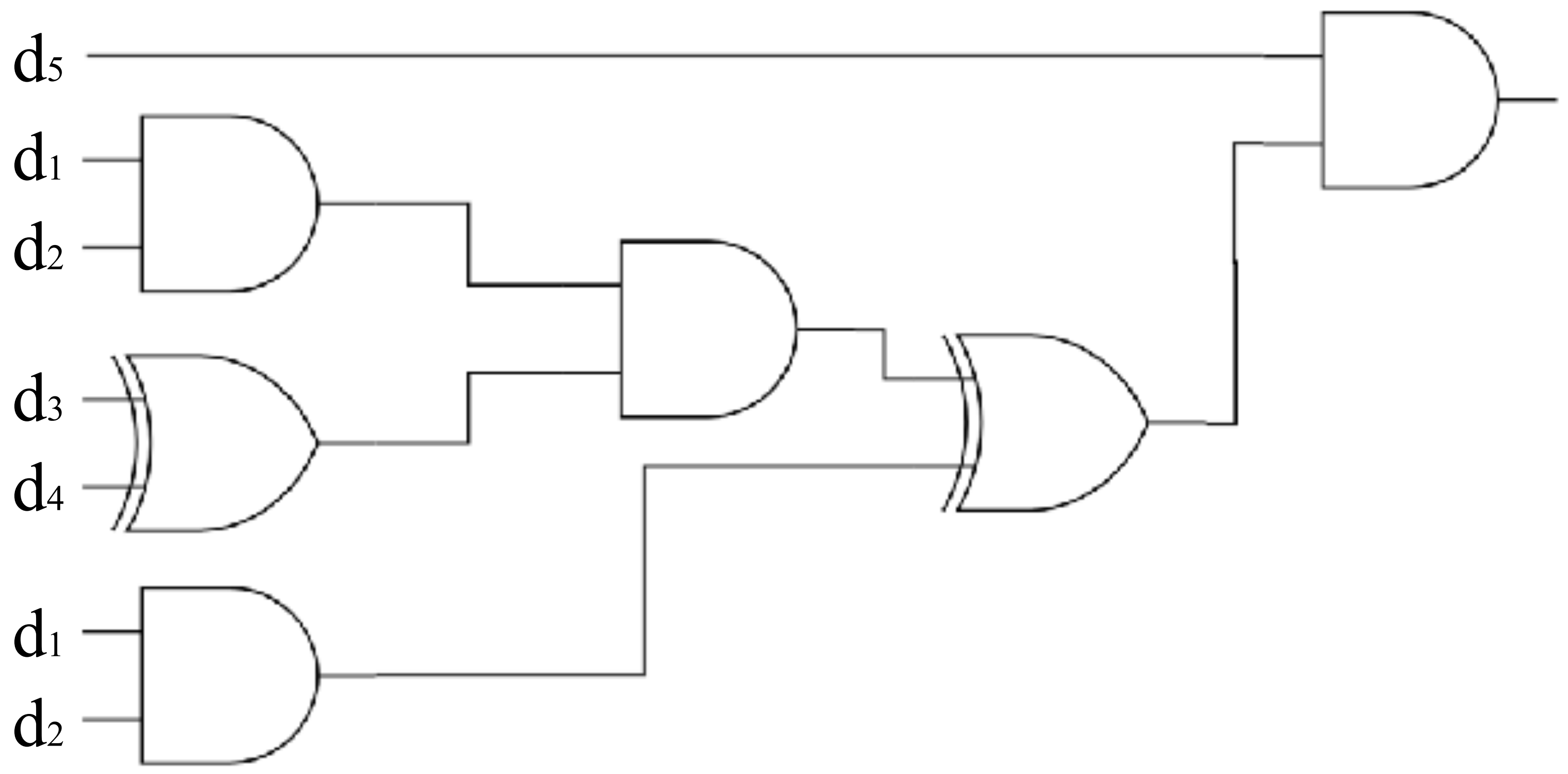
old



Normalized Opt. Patterns

new





target

New Input Circuit Optimization

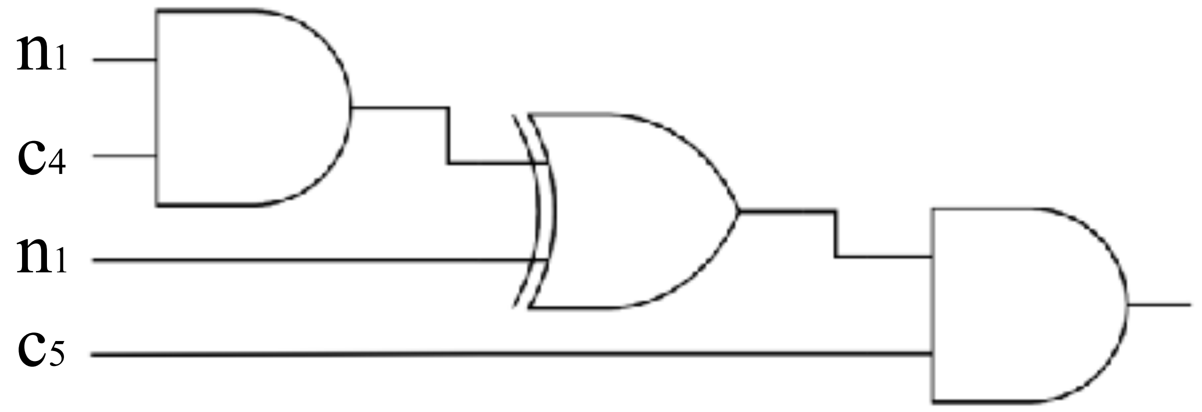
?

target'

Applying Learned Optimization Patterns (2/2)

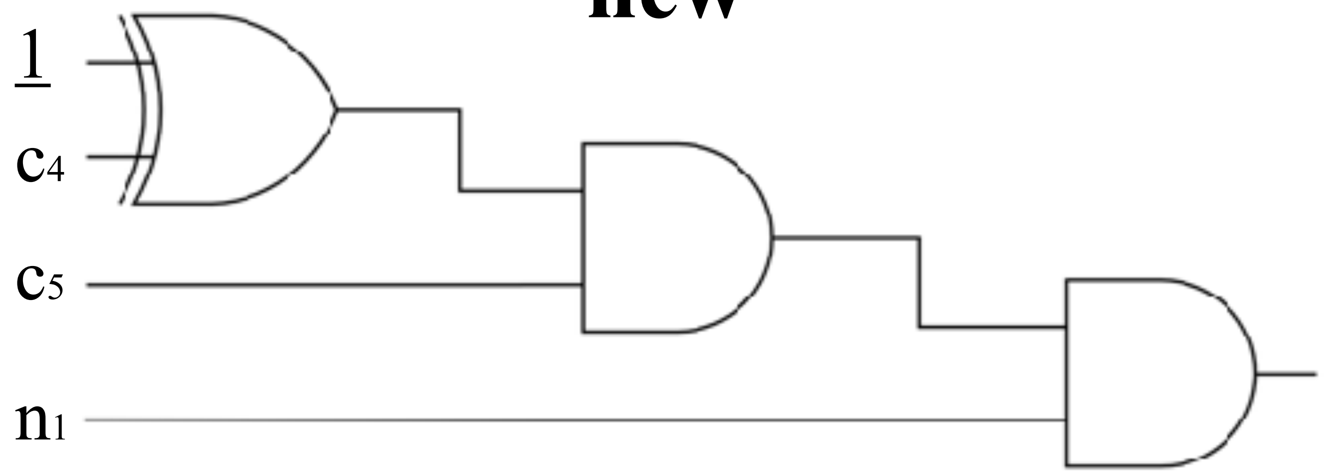
Normalization + Equational Matching

old



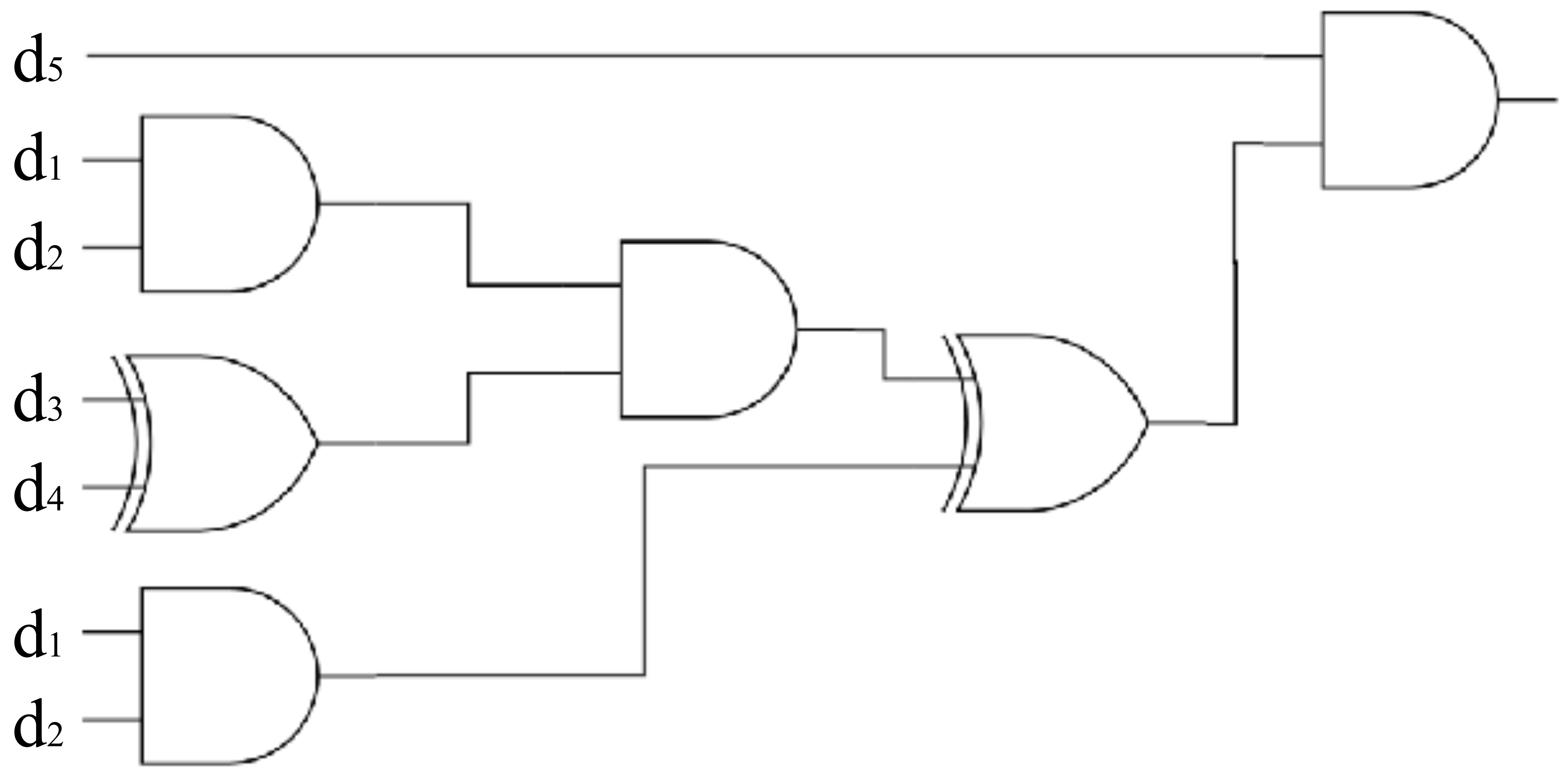
Normalized Opt. Patterns

new



Find substitution σ

(considering commutativity)



target

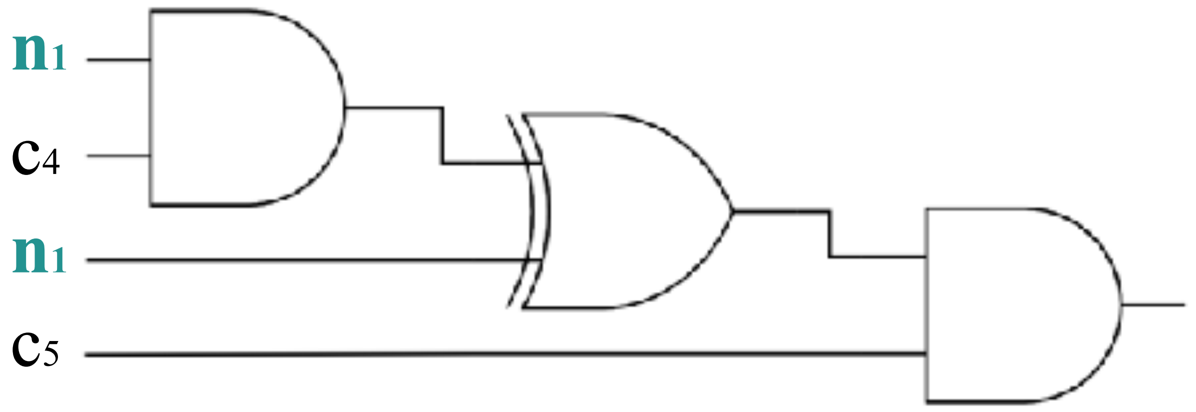
New Input Circuit Optimization

?

Applying Learned Optimization Patterns (2/2)

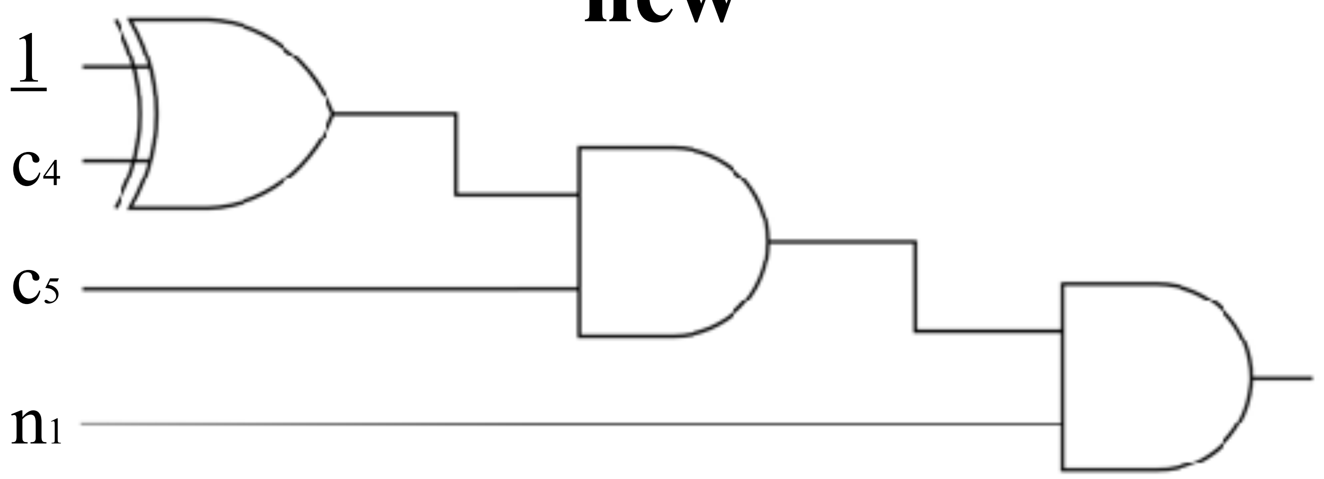
Normalization + Equational Matching

old



Normalized Opt. Patterns

new



Find substitution σ

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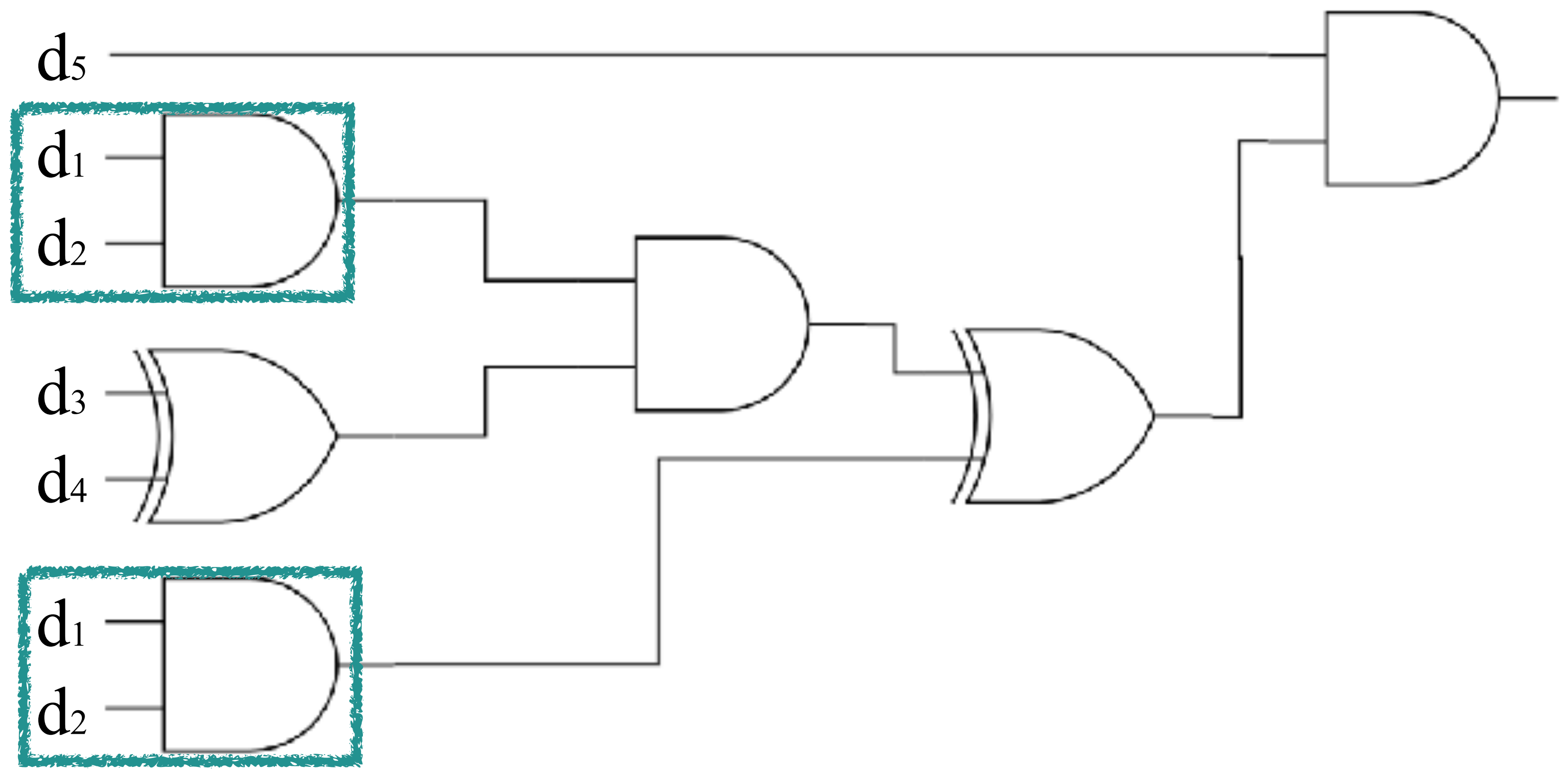
σ = {n1 ↦ d1 and d2,



target

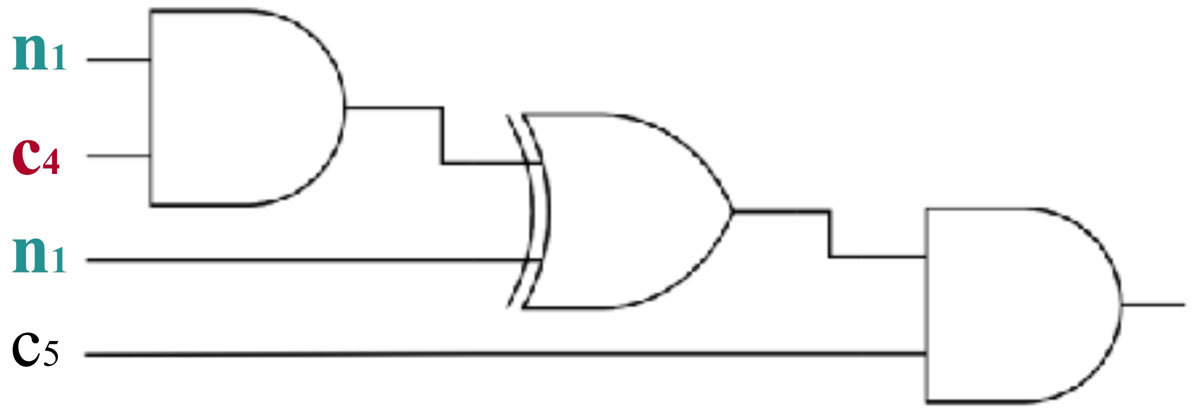
New Input Circuit Optimization

?

Applying Learned Optimization Patterns (2/2)

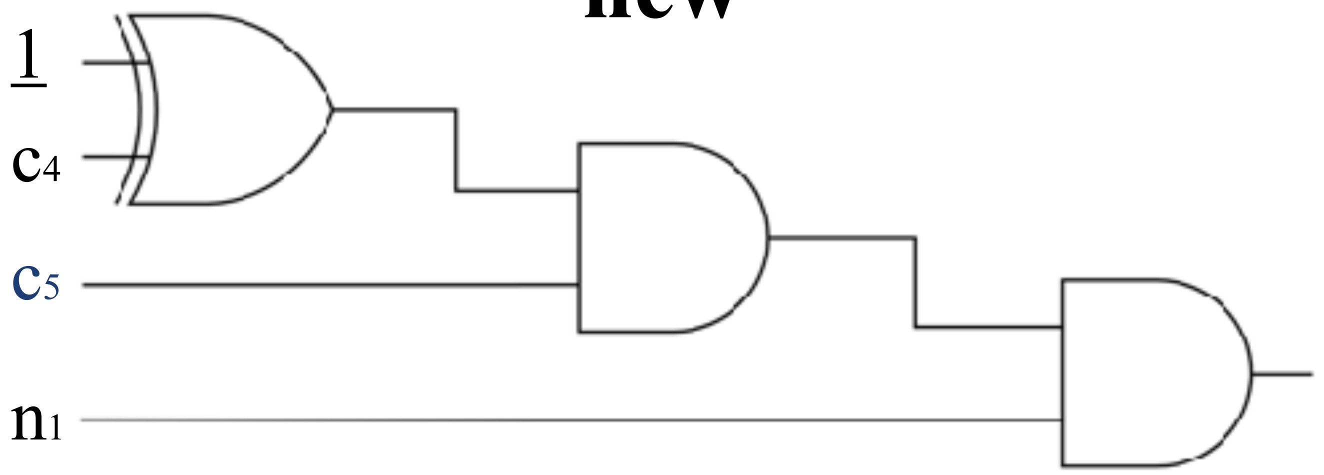
Normalization + Equational Matching

old



Normalized Opt. Patterns

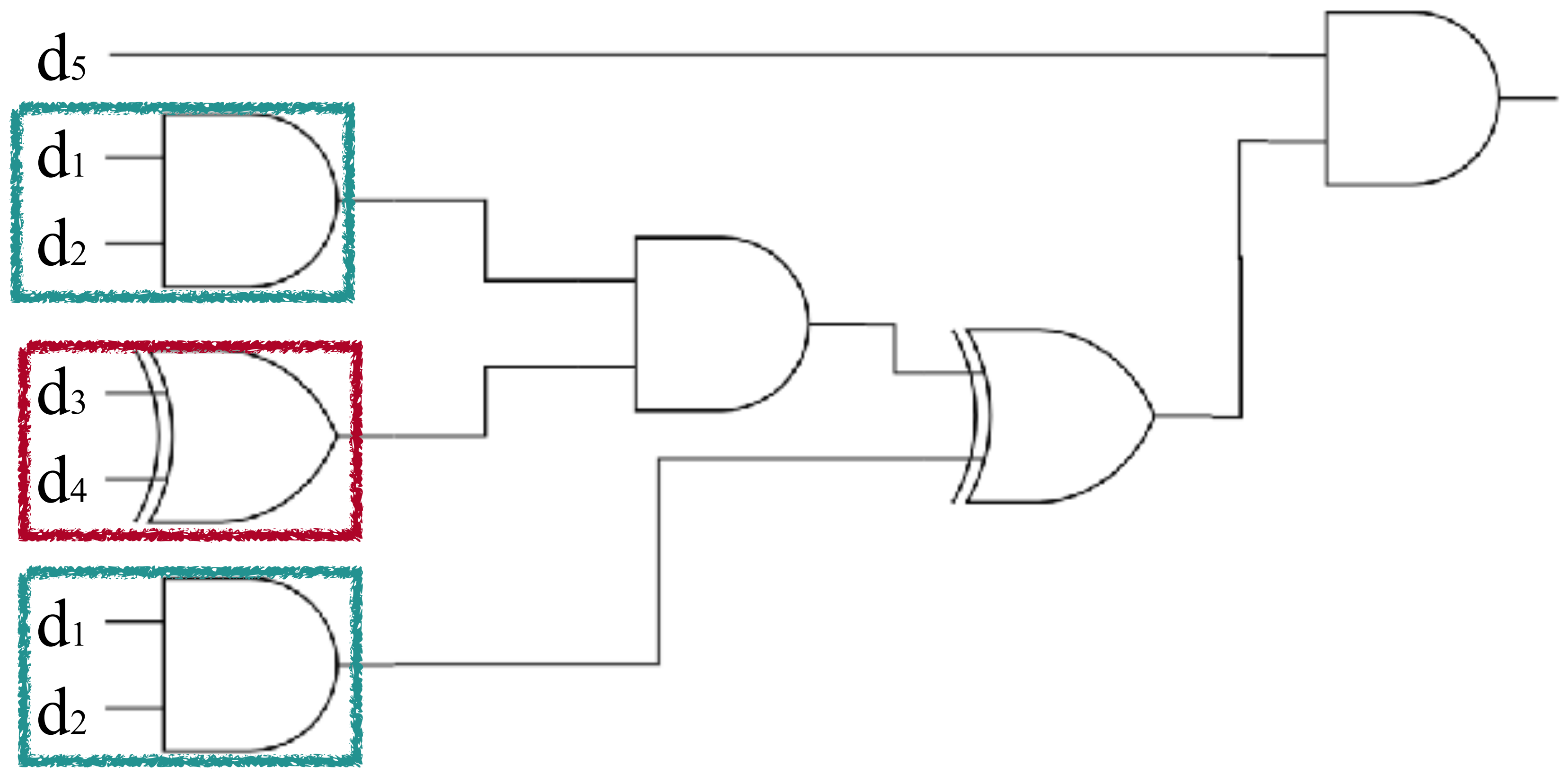
new



Find substitution σ

(considering commutativity)

σ = {n1 ↦ d1 and d2,
c4 ↦ d3 xor d4,



target

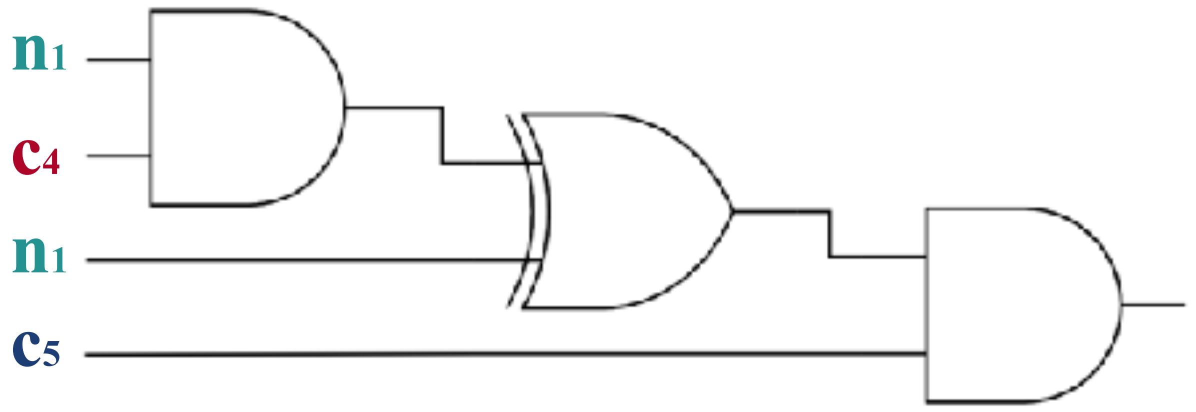
New Input Circuit Optimization

?

Applying Learned Optimization Patterns (2/2)

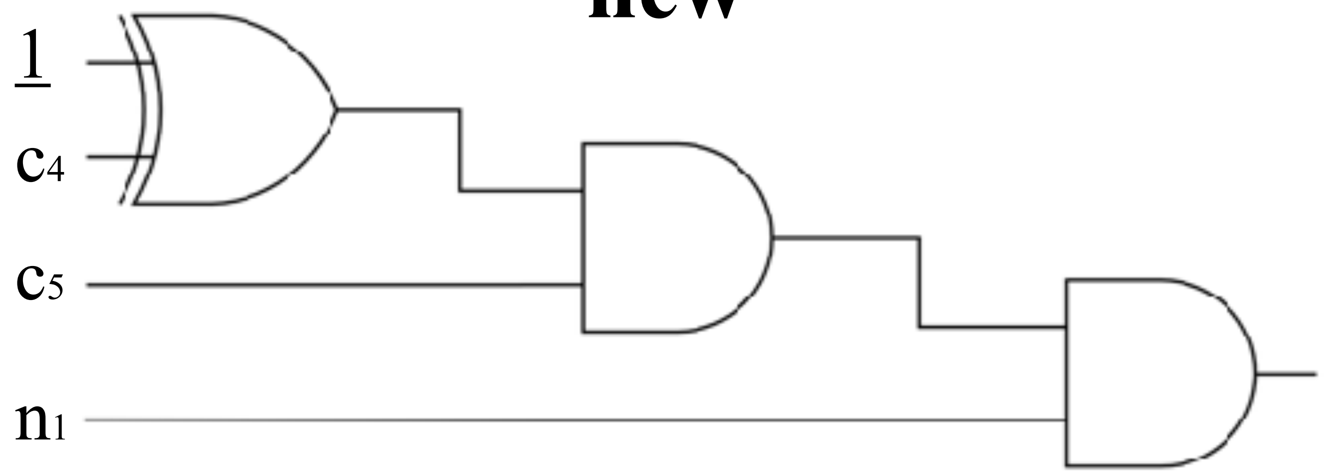
Normalization + Equational Matching

old



Normalized Opt. Patterns

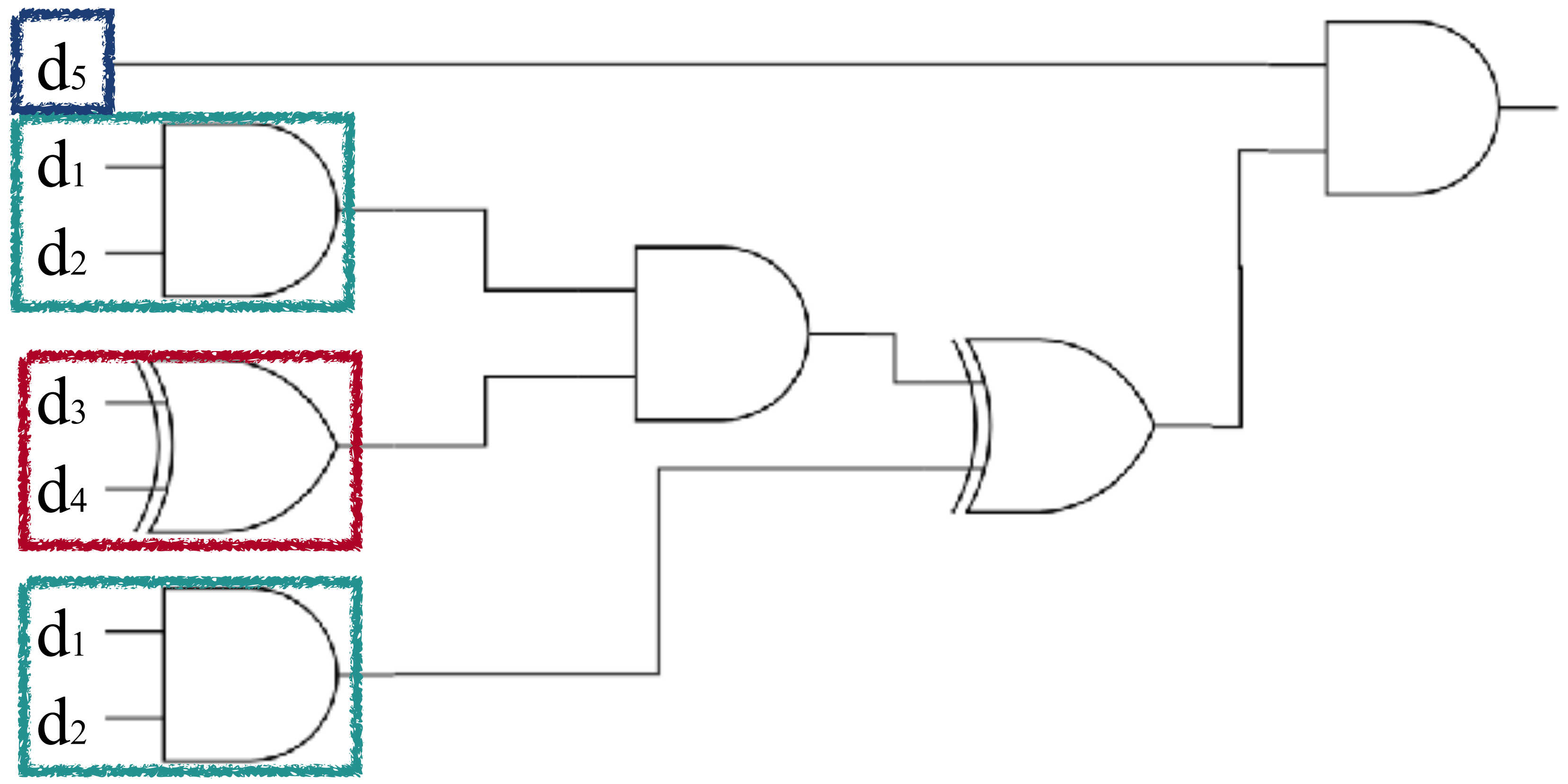
new



Find substitution σ

(considering commutativity)

σ = {n1 ↦ d1 and d2,
c4 ↦ d3 xor d4,
c5 ↦ d5}



target

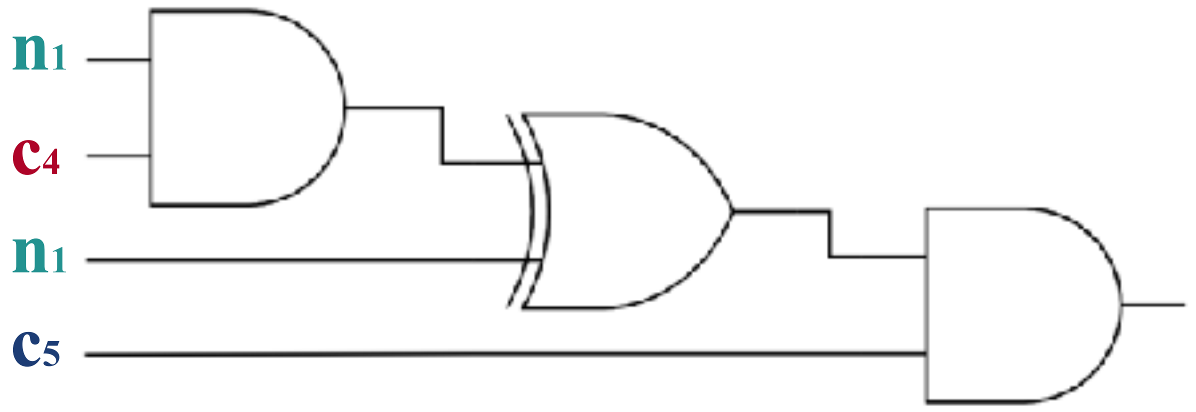
New Input Circuit Optimization

?

Applying Learned Optimization Patterns (2/2)

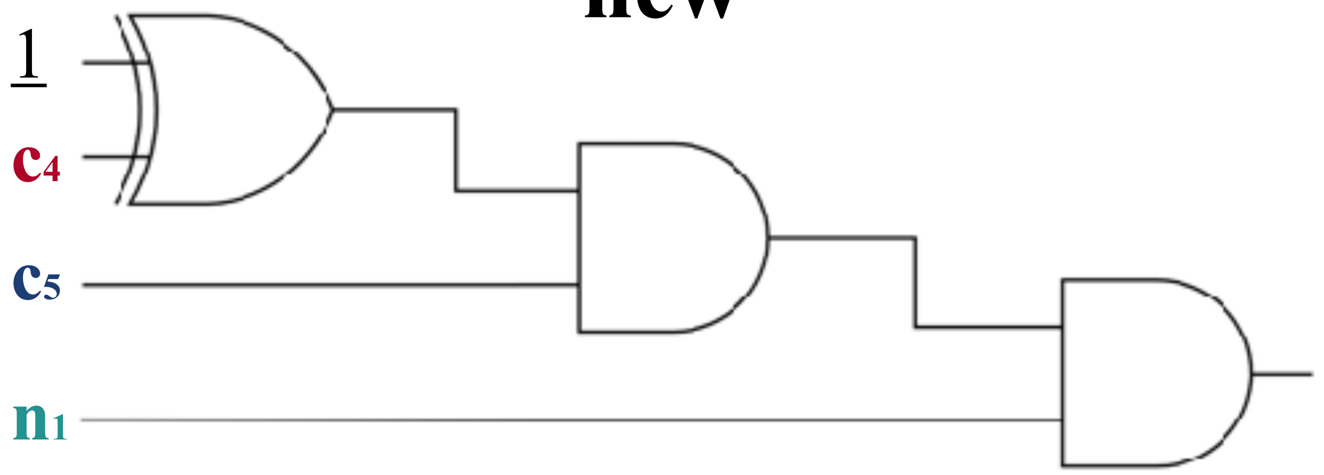
Normalization + Equational Matching

old



Normalized
Opt. Patterns

new

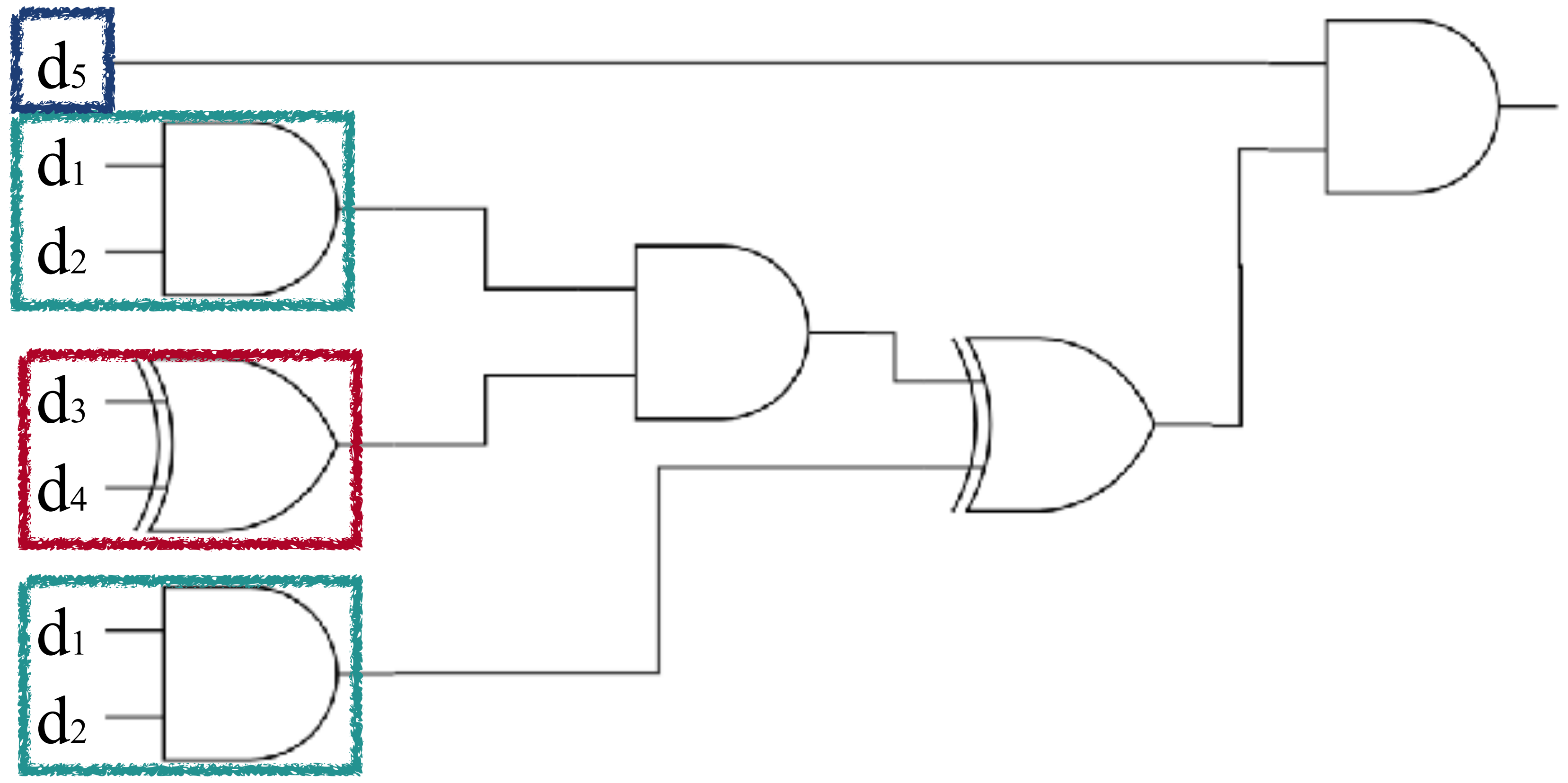


Find substitution σ

(considering commutativity)

Apply substitution σ

σ = {n1 ↦ d1 and d2,
c4 ↦ d3 xor d4,
c5 ↦ d5}



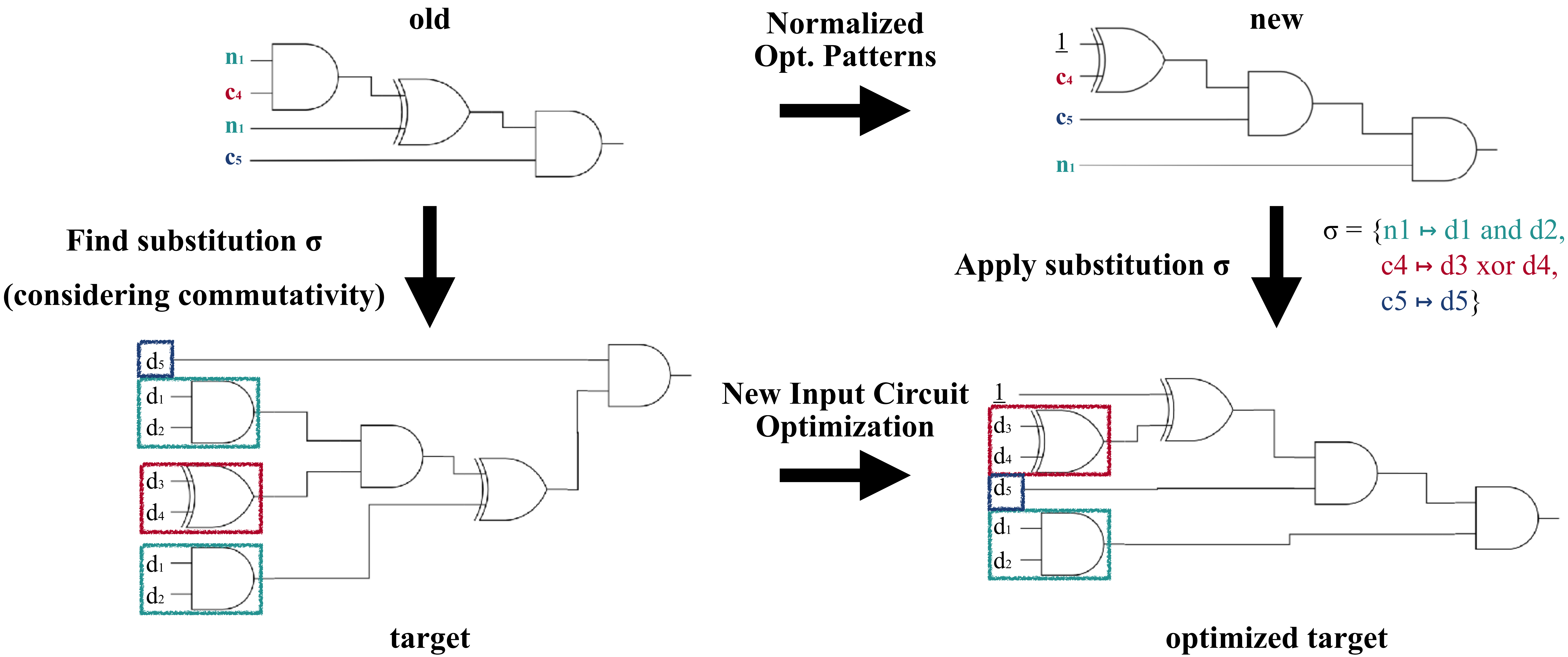
target

New Input Circuit
Optimization

?

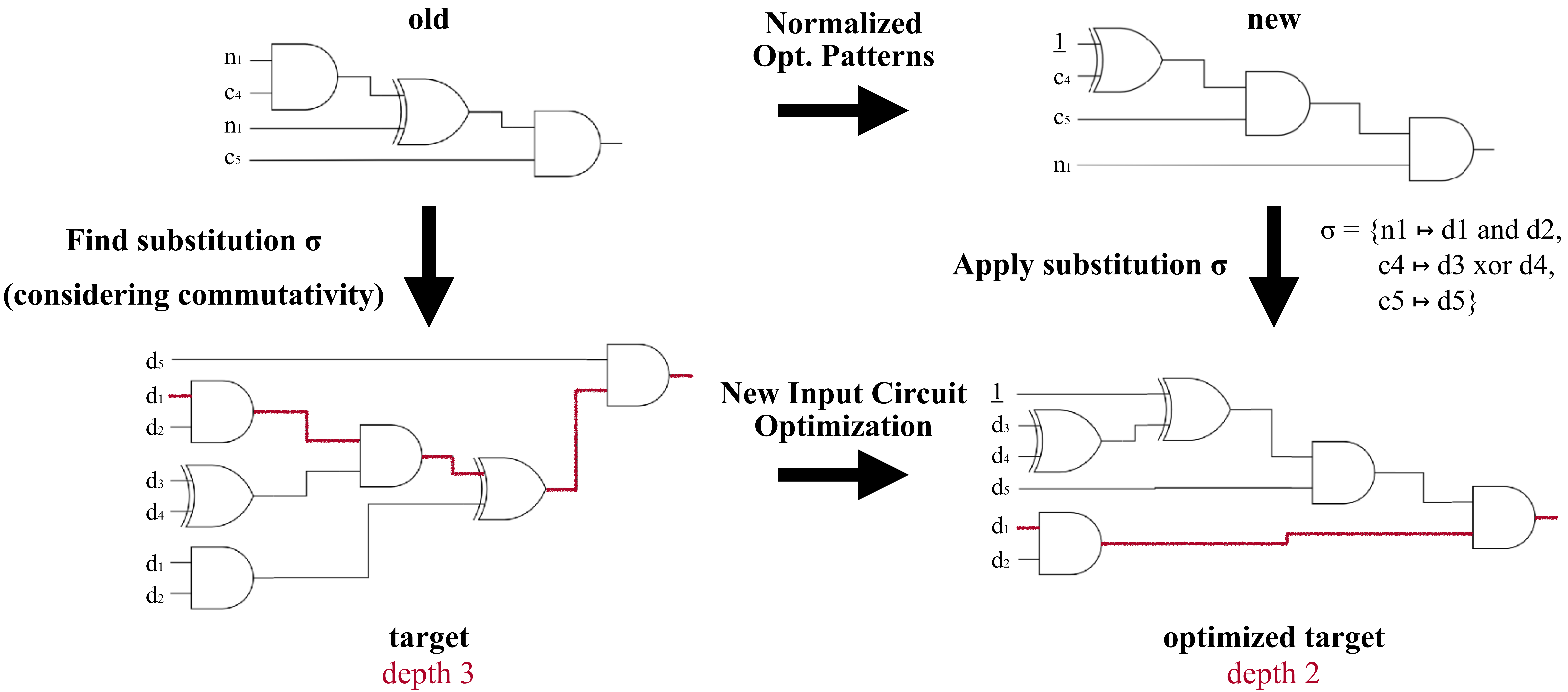
Applying Learned Optimization Patterns (2/2)

Normalization + Equational Matching



Applying Learned Optimization Patterns (2/2)

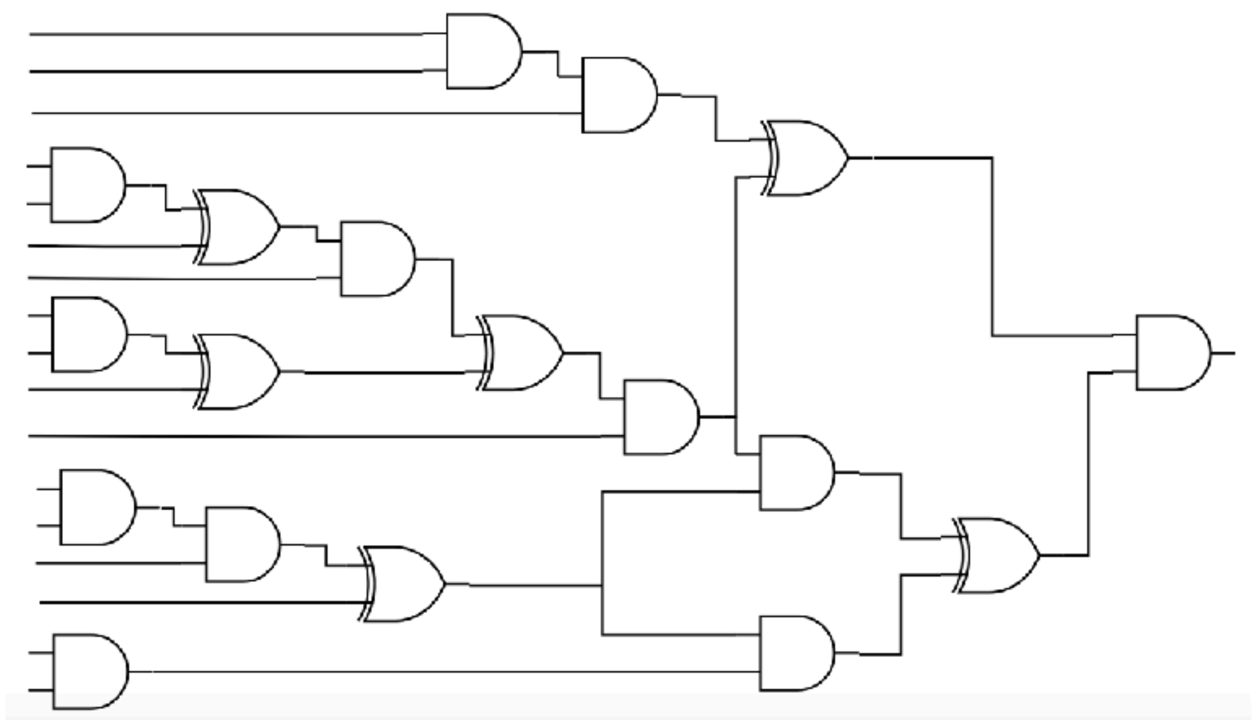
Normalization + Equational Matching



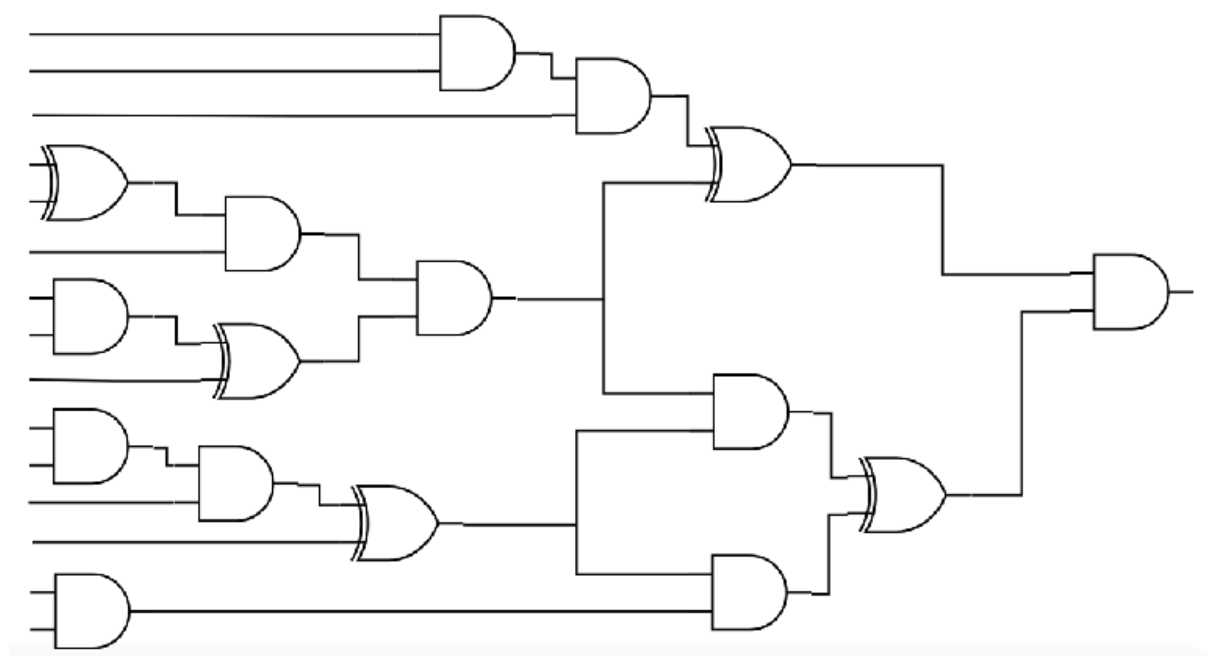
Applying Learned Optimization Patterns

Formal properties

(Soundness) semantics unchanged



Applying an opt. pattern



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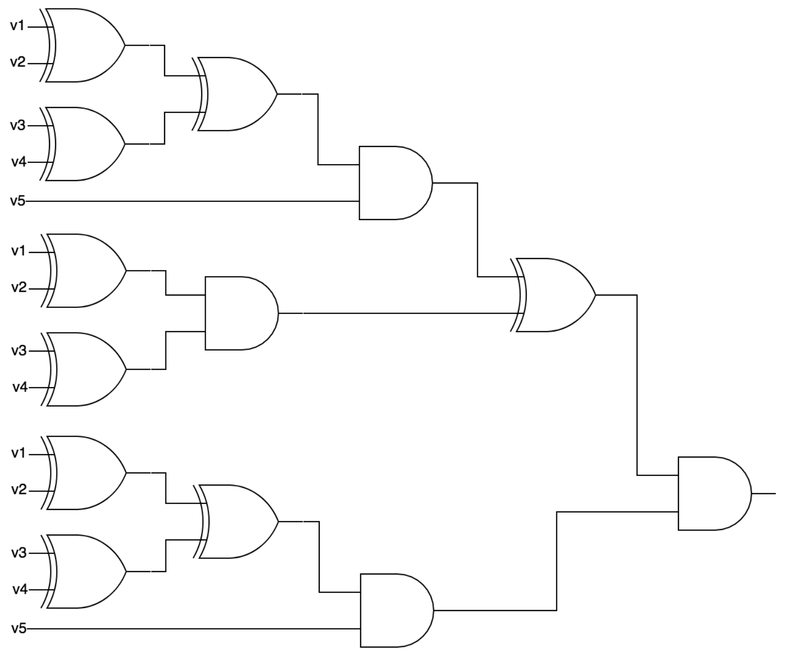


(Termination) finitely many rule applications

Lobster Performance (1/5)

Benchmarks

- 25 HE algorithms from 4 sources
  - Cingulata benchmarks
  - Sorting benchmarks
  - Hackers Delight benchmarks
  - EPFL benchmarks

2 HE friendly algorithms (medical, sorting)

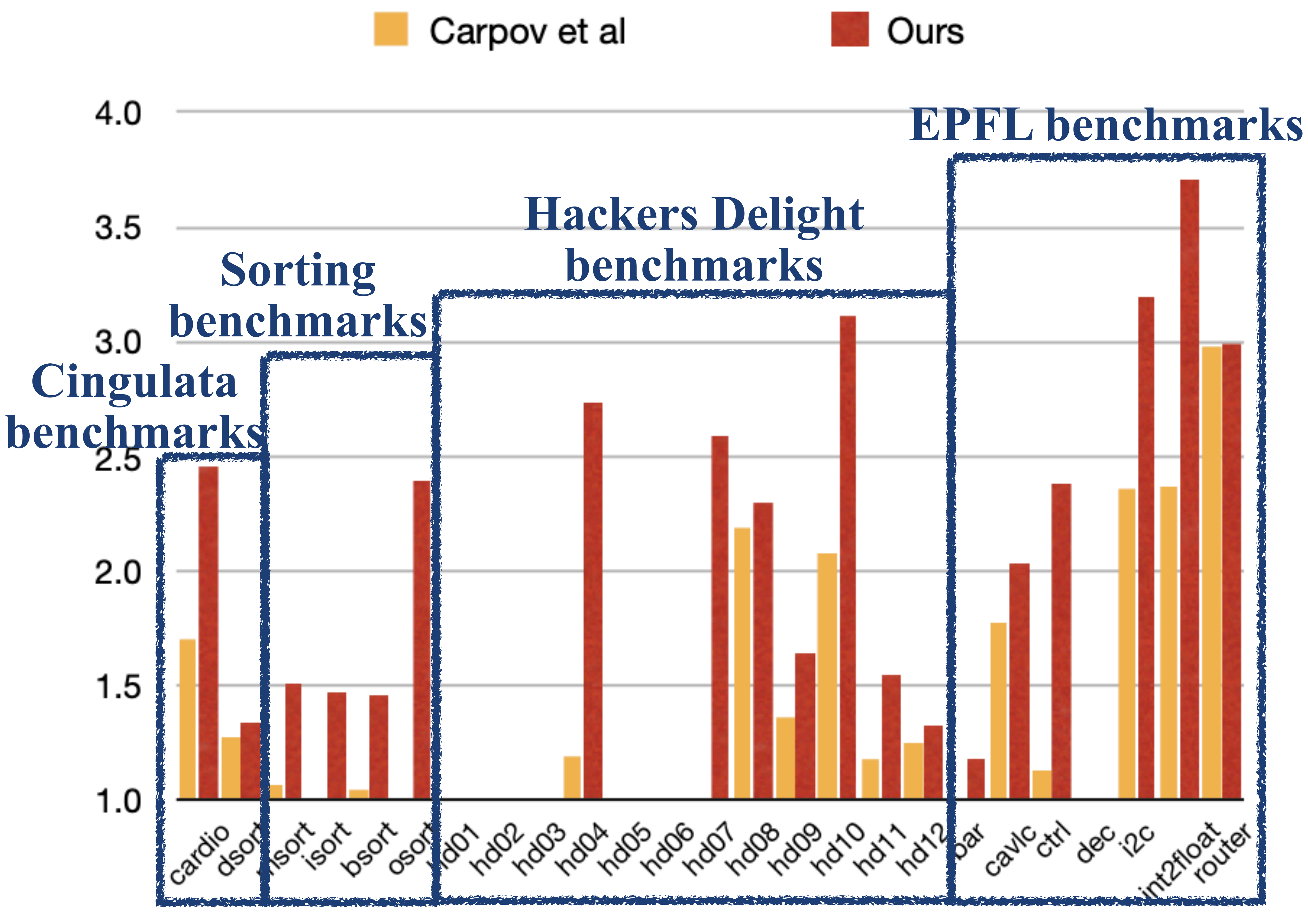
4 privacy-preserving sorting algorithms (merge, insert, bubble, odd-even)

12 Homomorphic bitwise operations

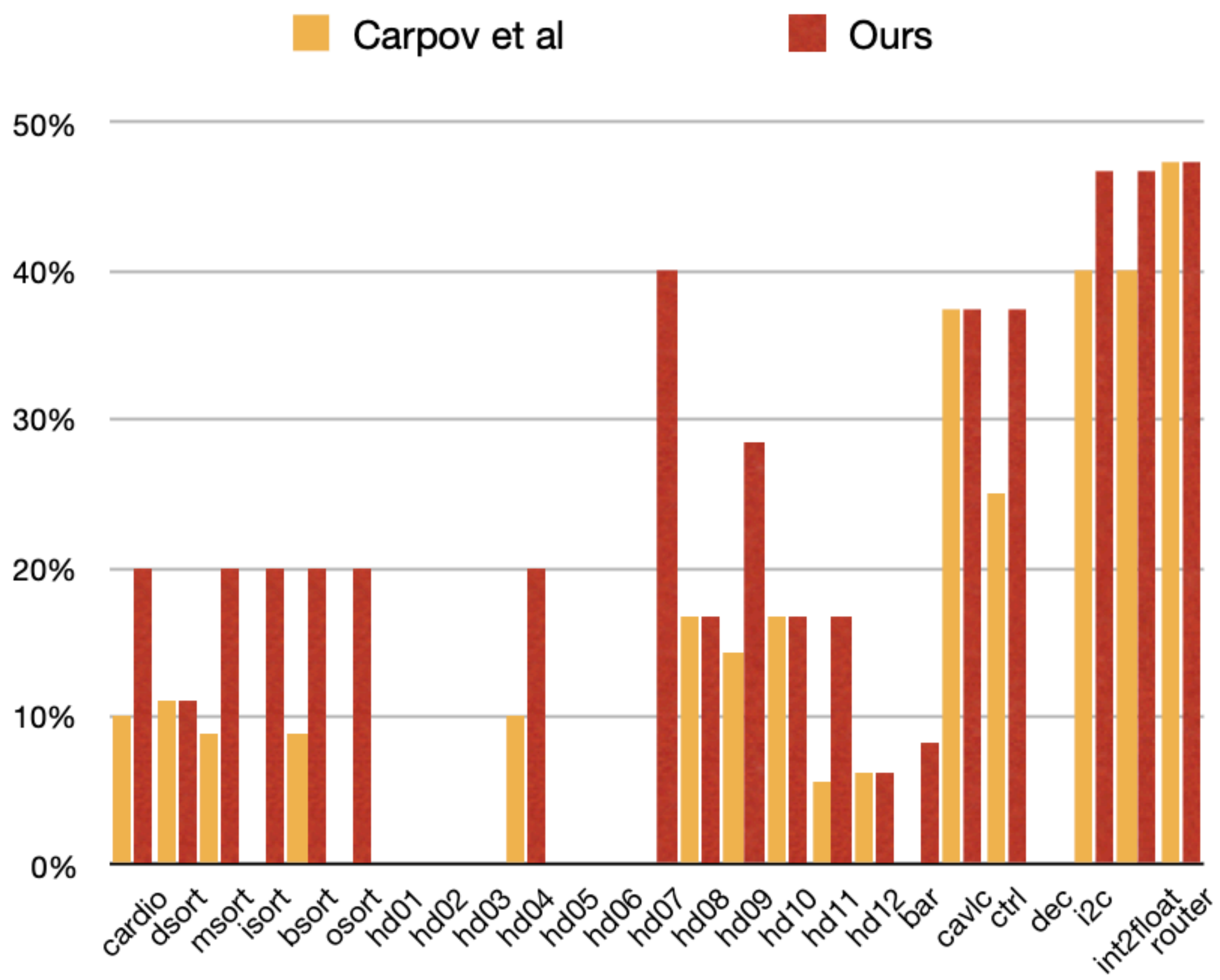
7 EPFL combinational benchmark suite (to test circuit optimizer)

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



Speedup

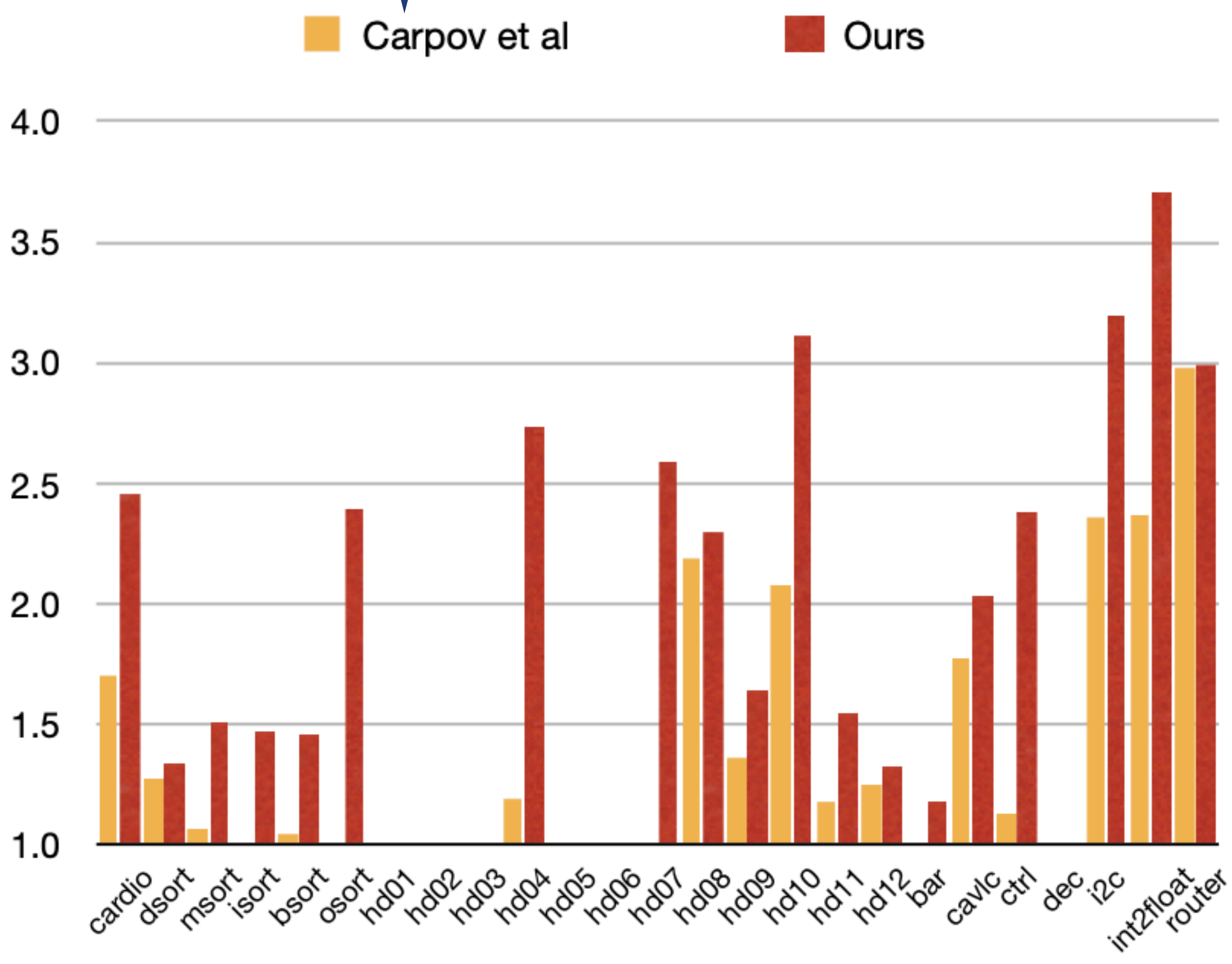


Depth Reduction

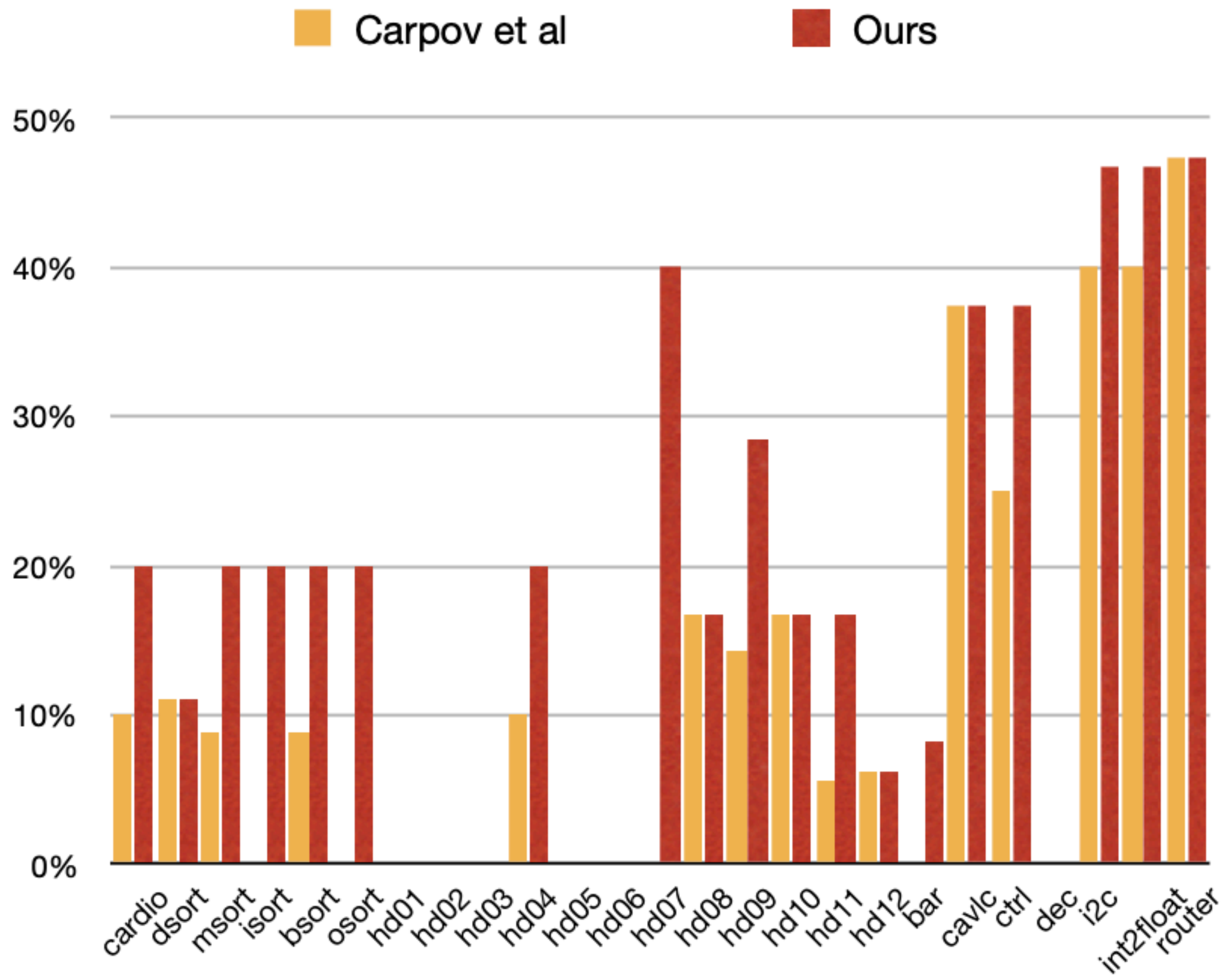
Lobster Performance (2/5)

Hand-written-rule based HE circuit optimizer

Optimization Results of Lobster and the baseline



Speedup



Depth Reduction

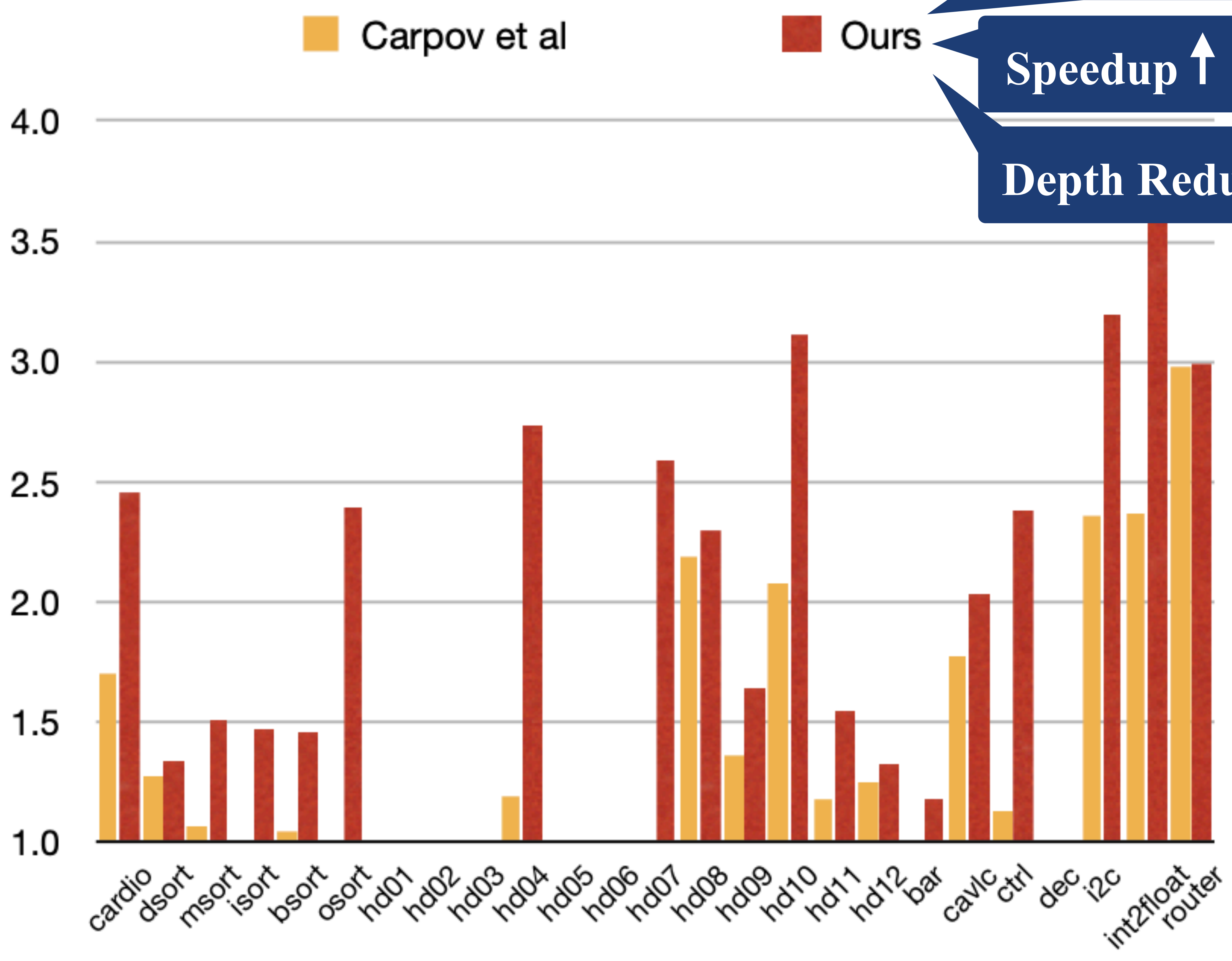
Lobster Performance (2/5)

Optimization Results of Lobster and the baseline

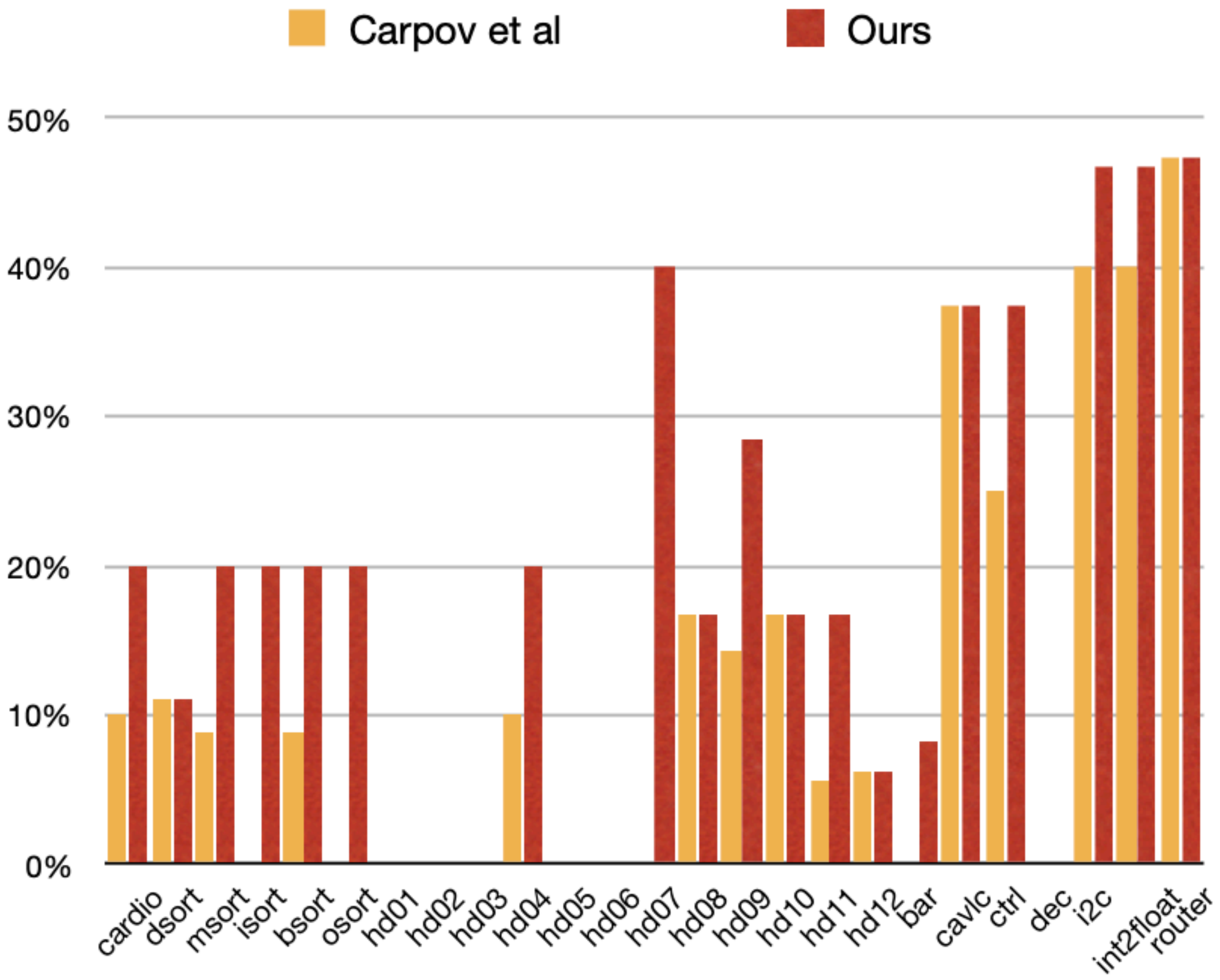
Success rate ↑

Speedup ↑

Depth Reduction ↑



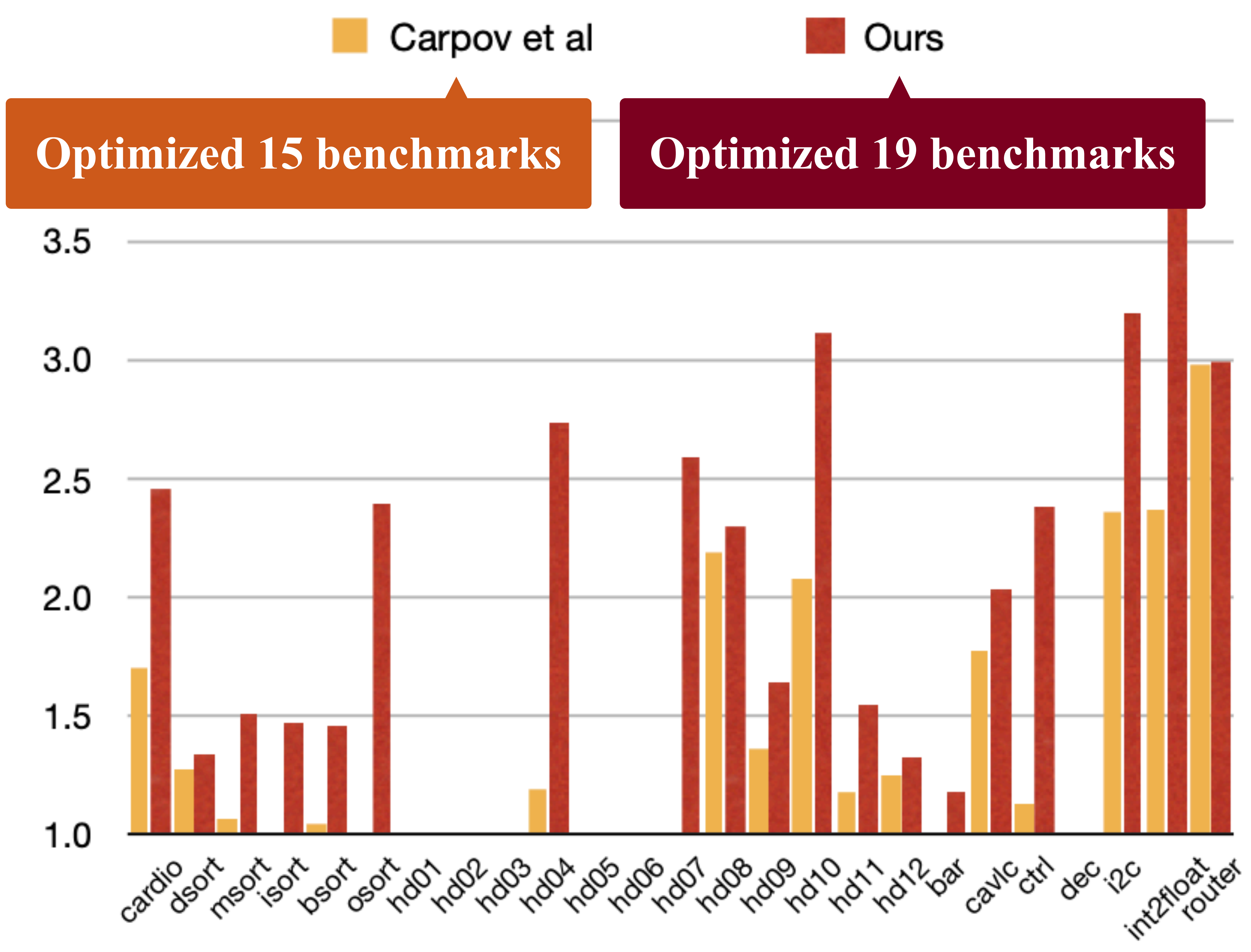
Speedup



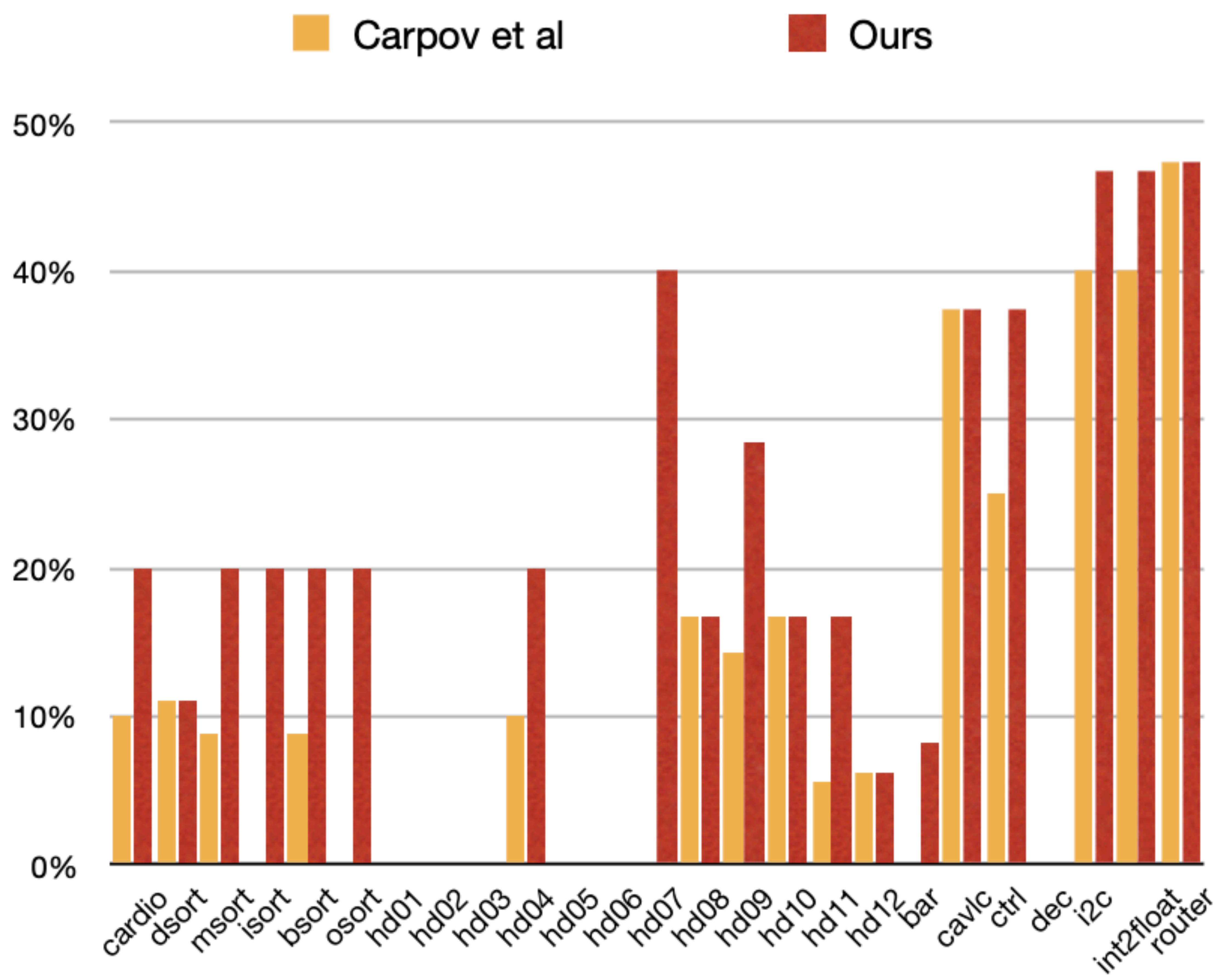
Depth Reduction

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



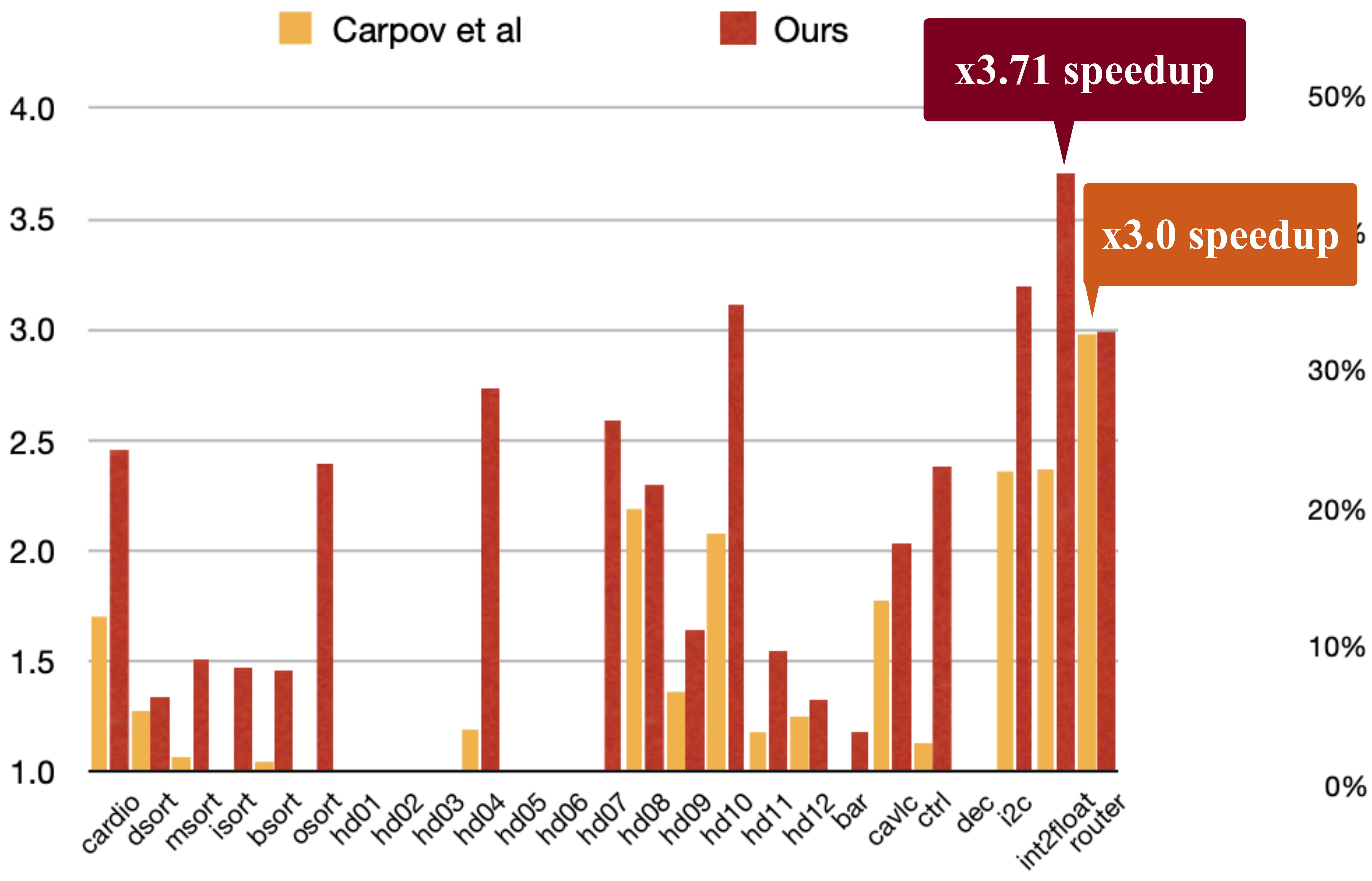
Speedup



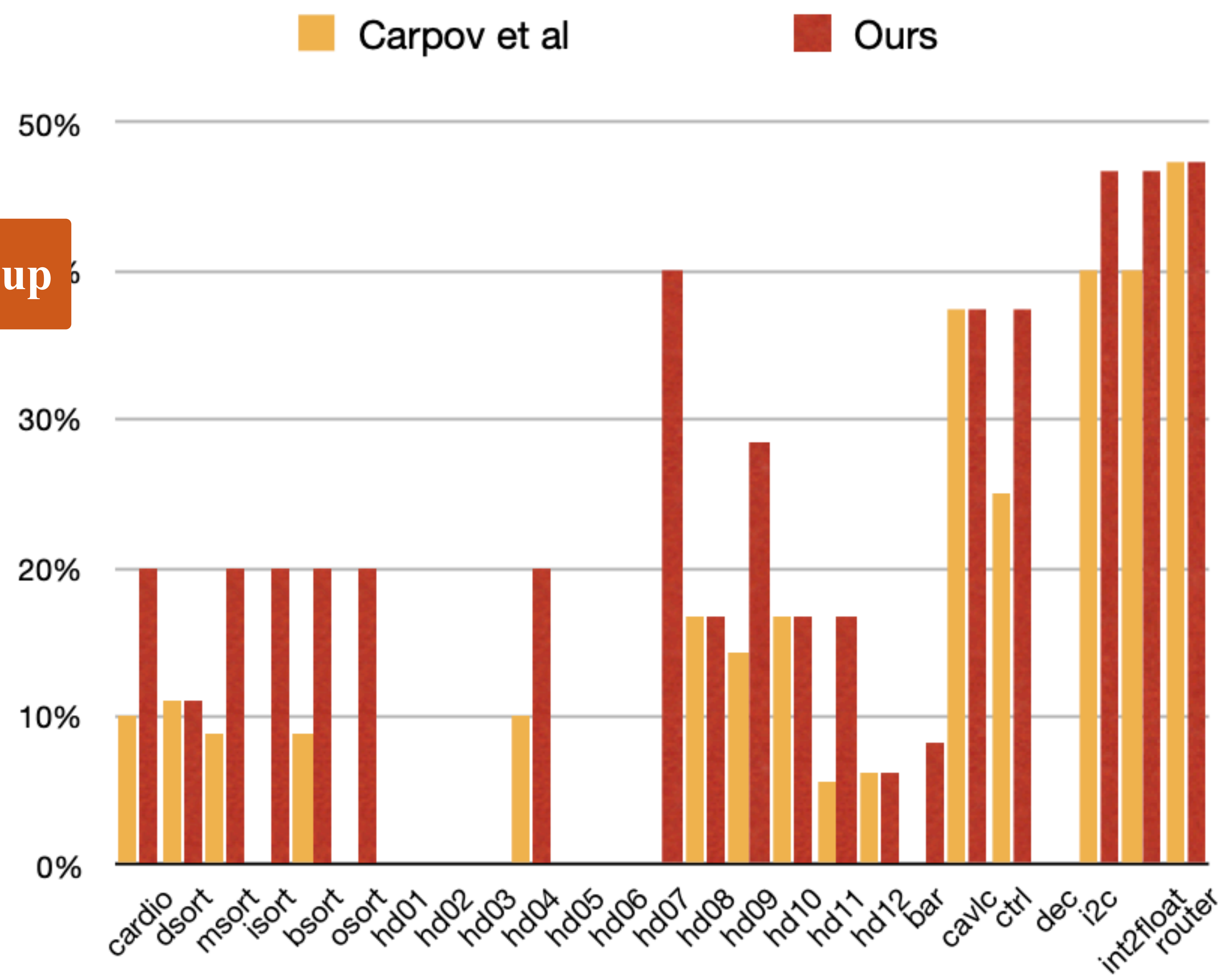
Depth Reduction

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



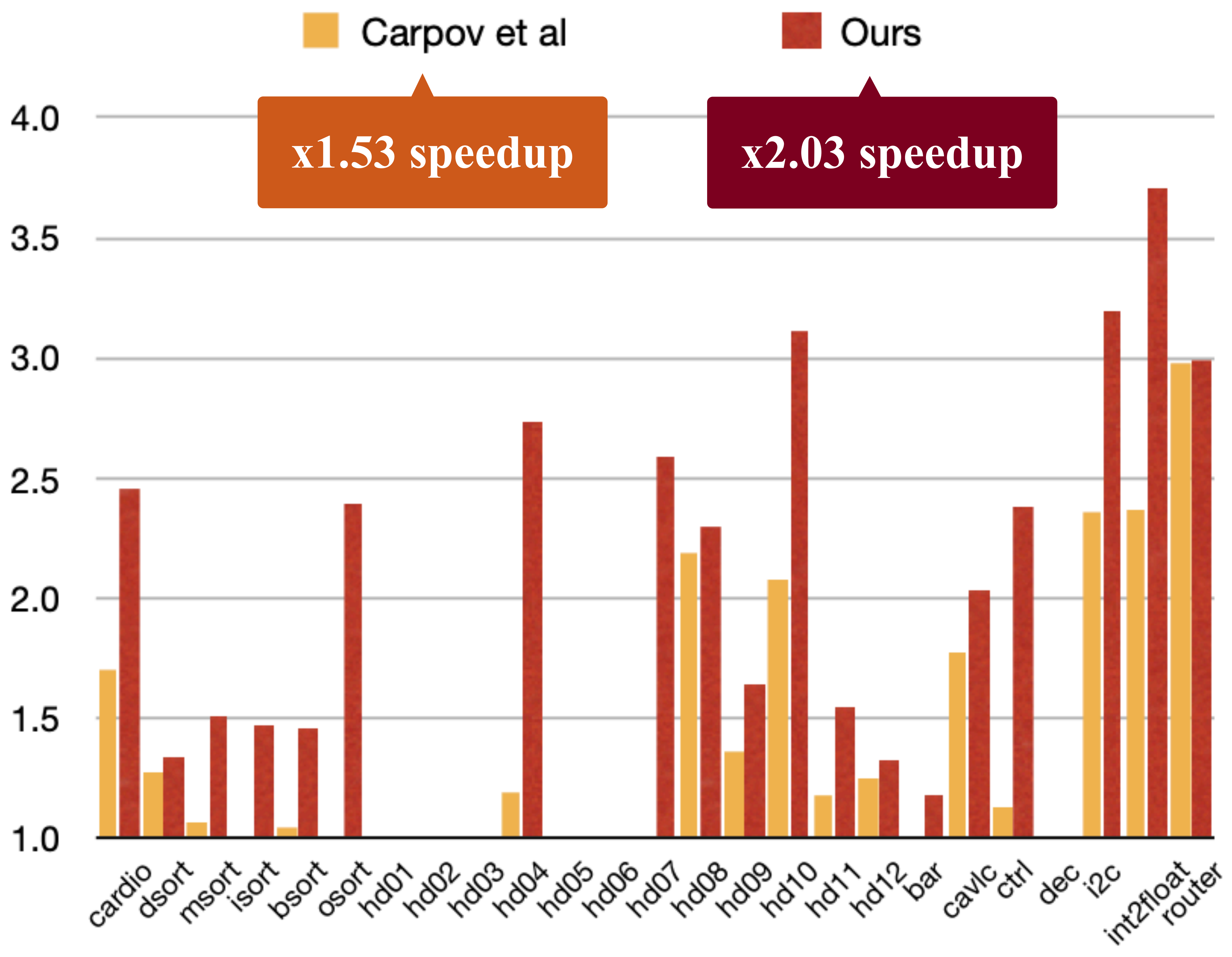
Speedup



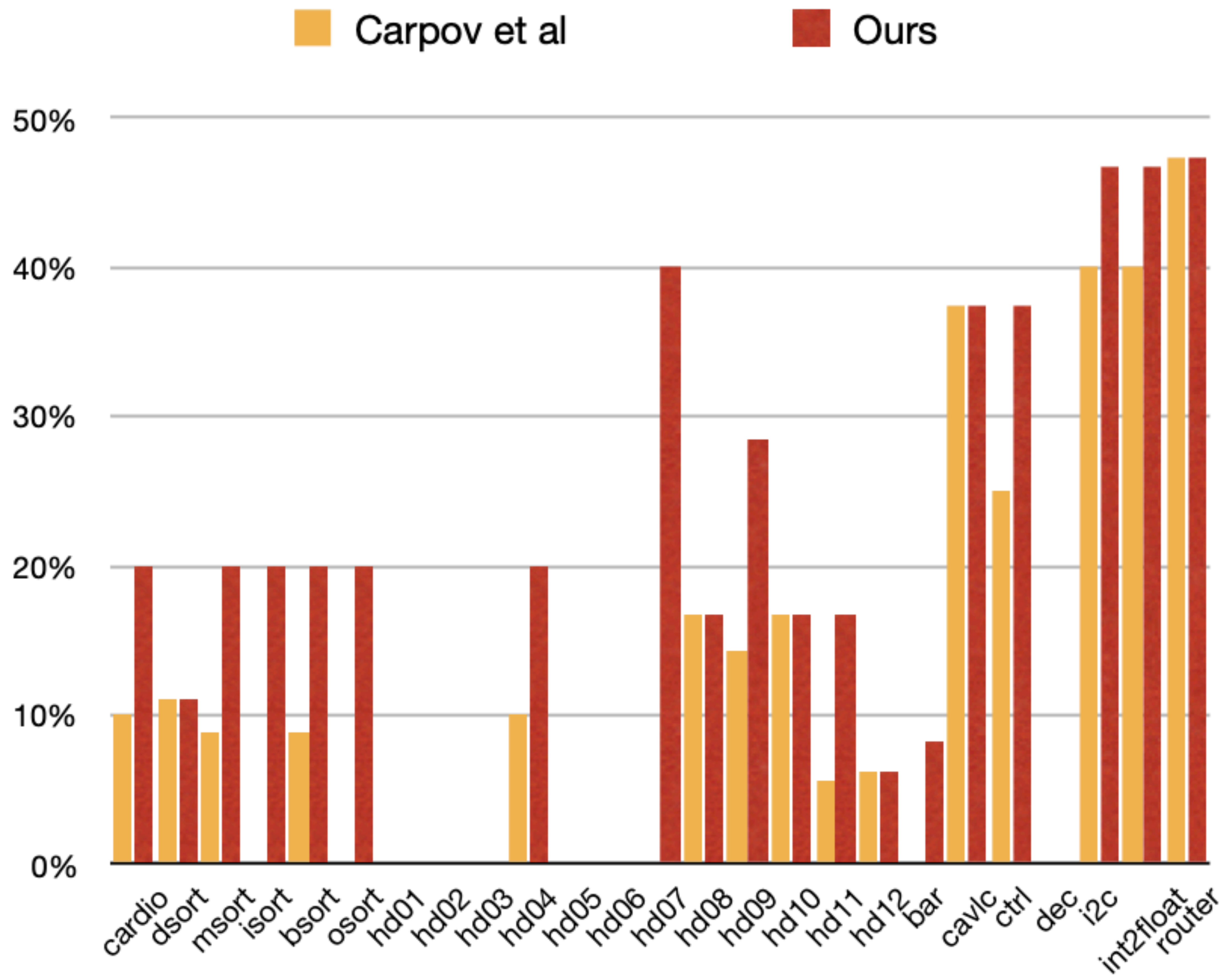
Depth Reduction

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



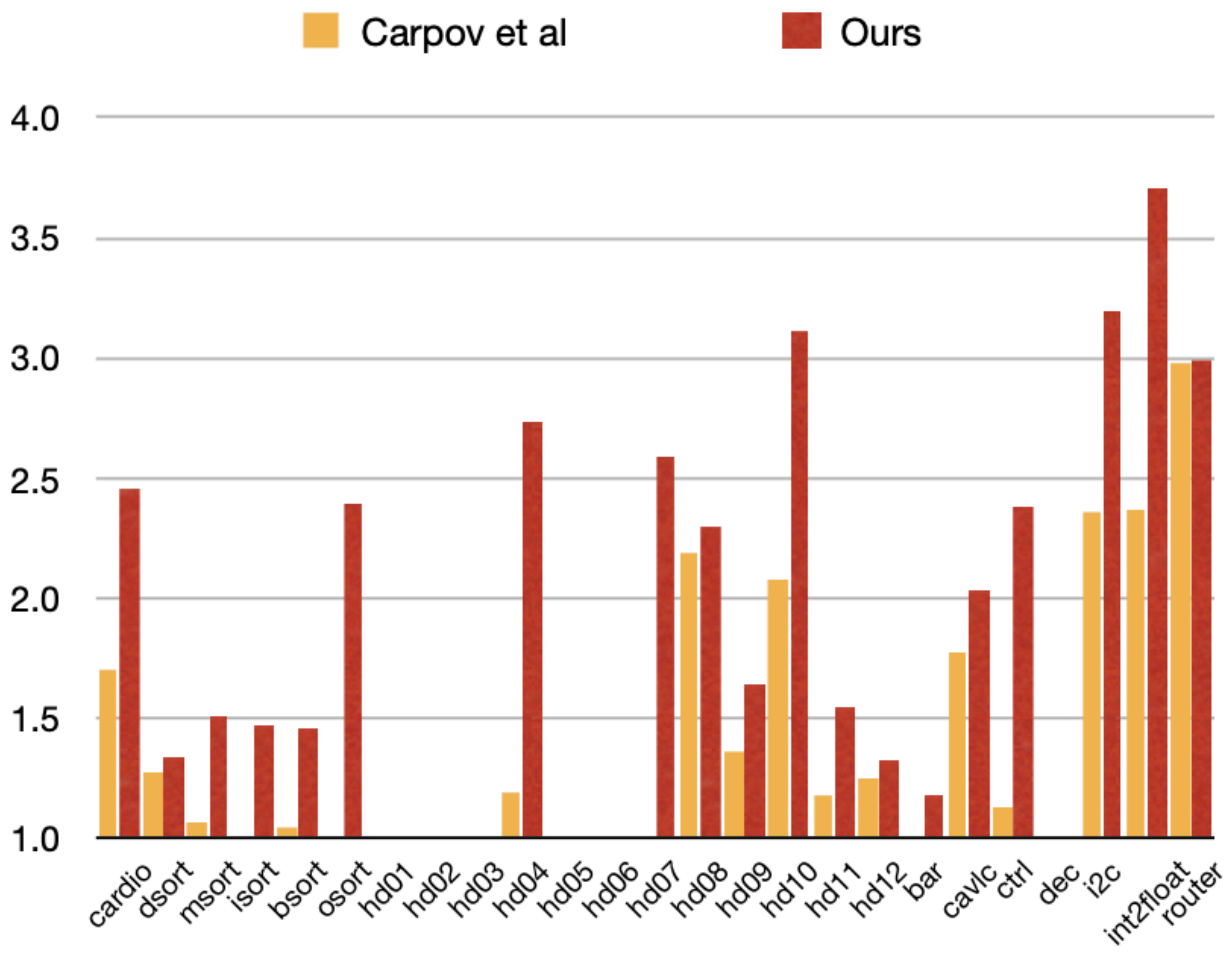
Speedup



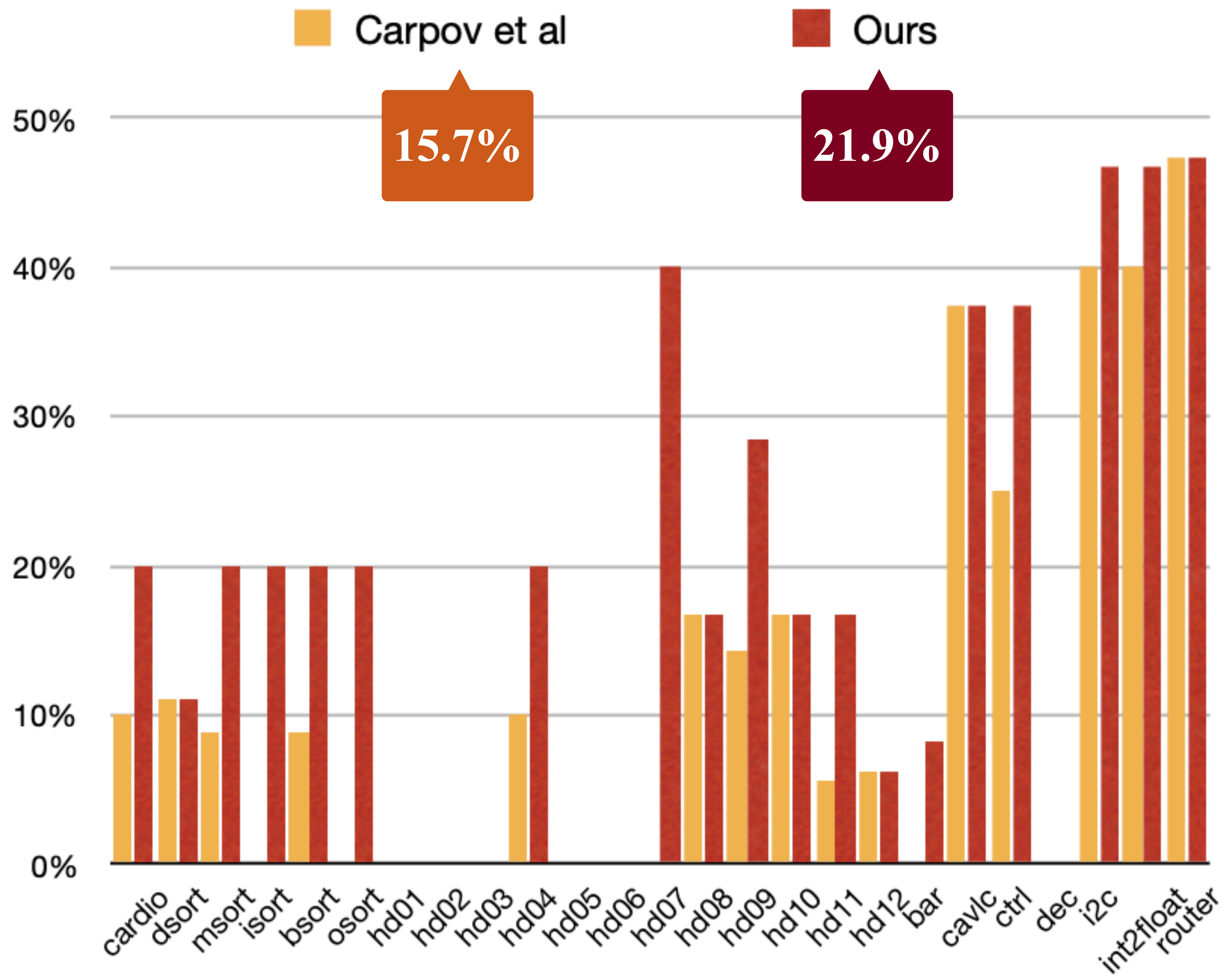
Depth Reduction

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



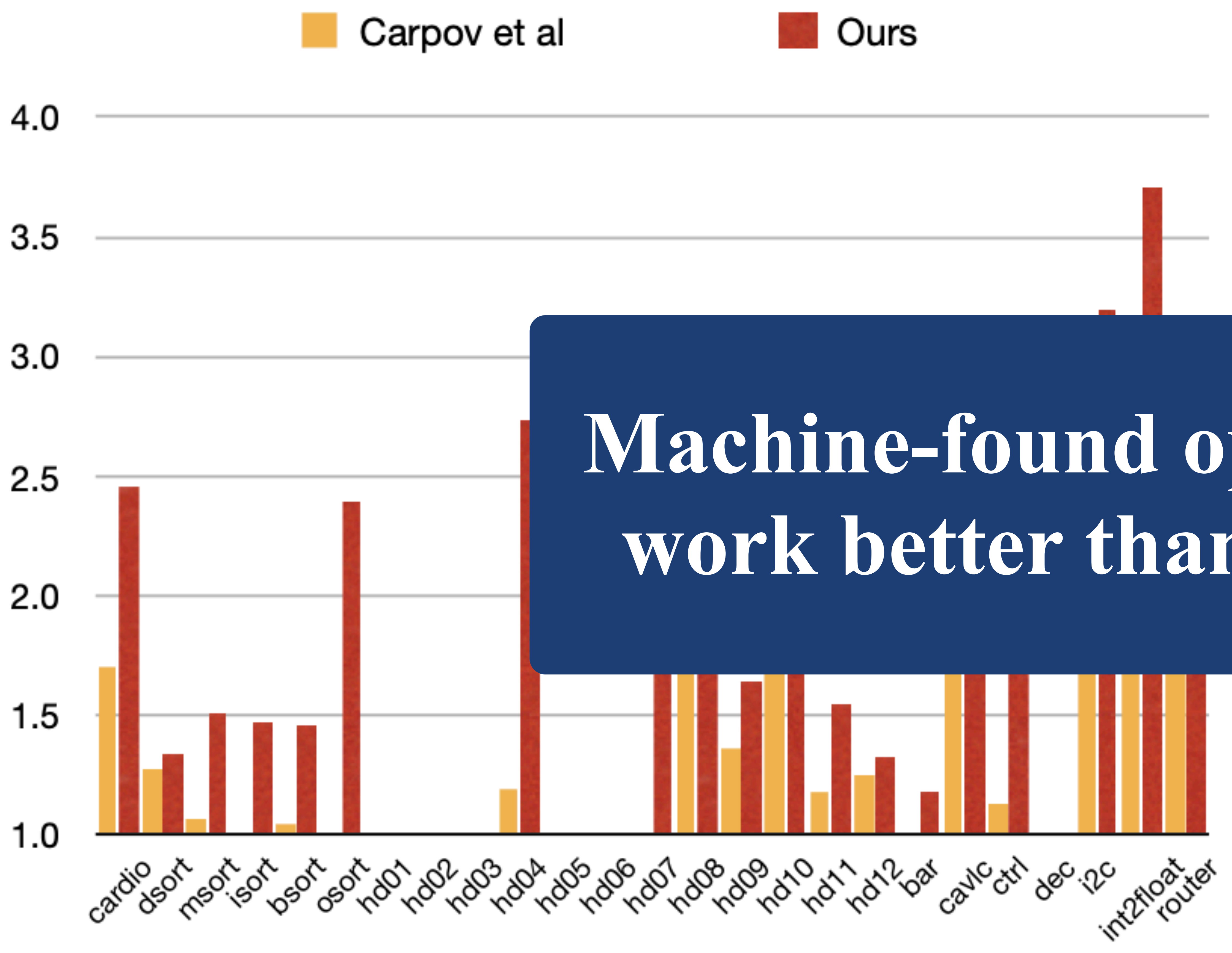
Speedup



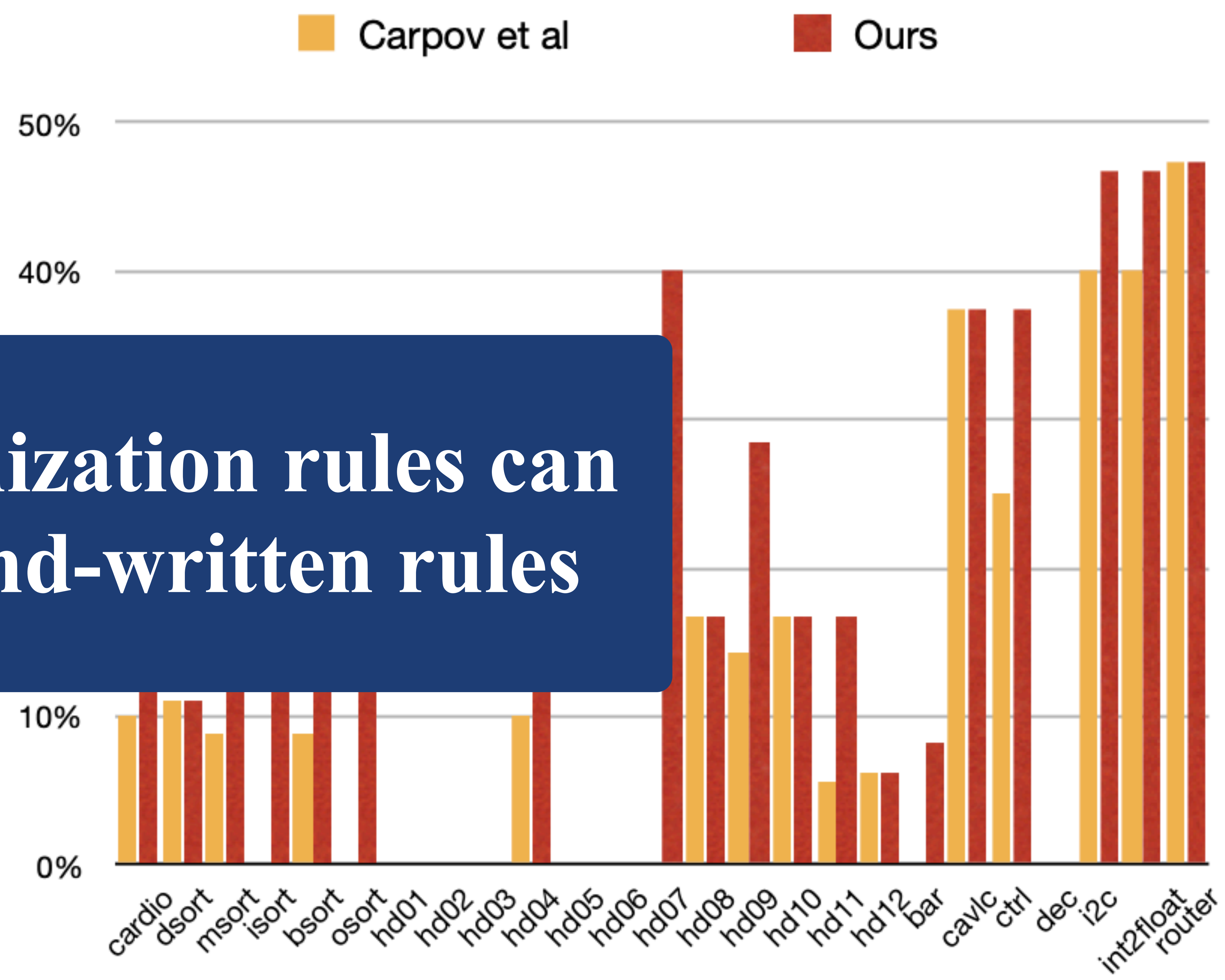
Depth Reduction

Lobster Performance (2/5)

Optimization Results of Lobster and the baseline



Speedup

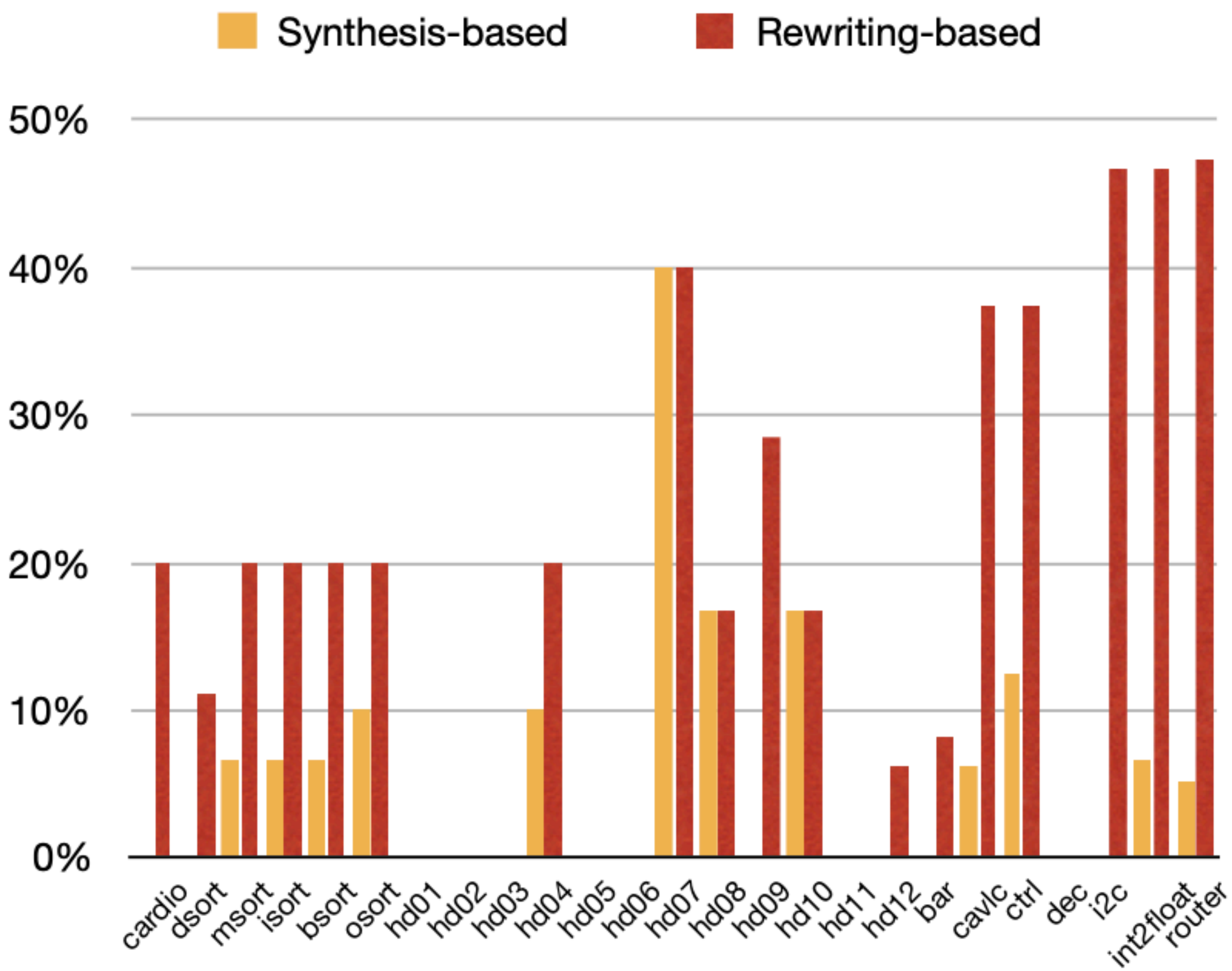


Depth Reduction

Machine-found optimization rules can work better than hand-written rules

Lobster Performance (3/5)

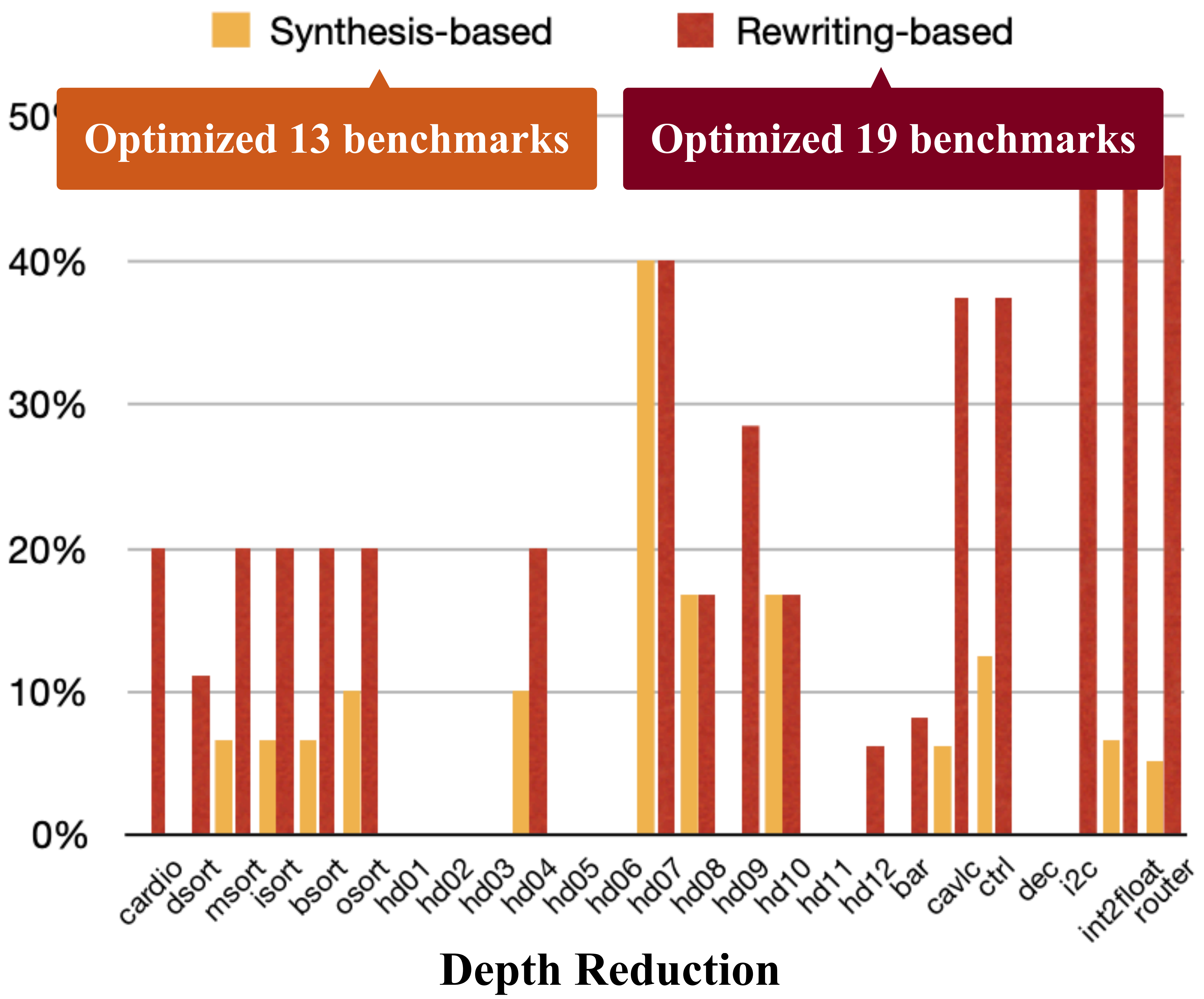
Efficacy of Reusing Learned Optimization Patterns



Depth Reduction

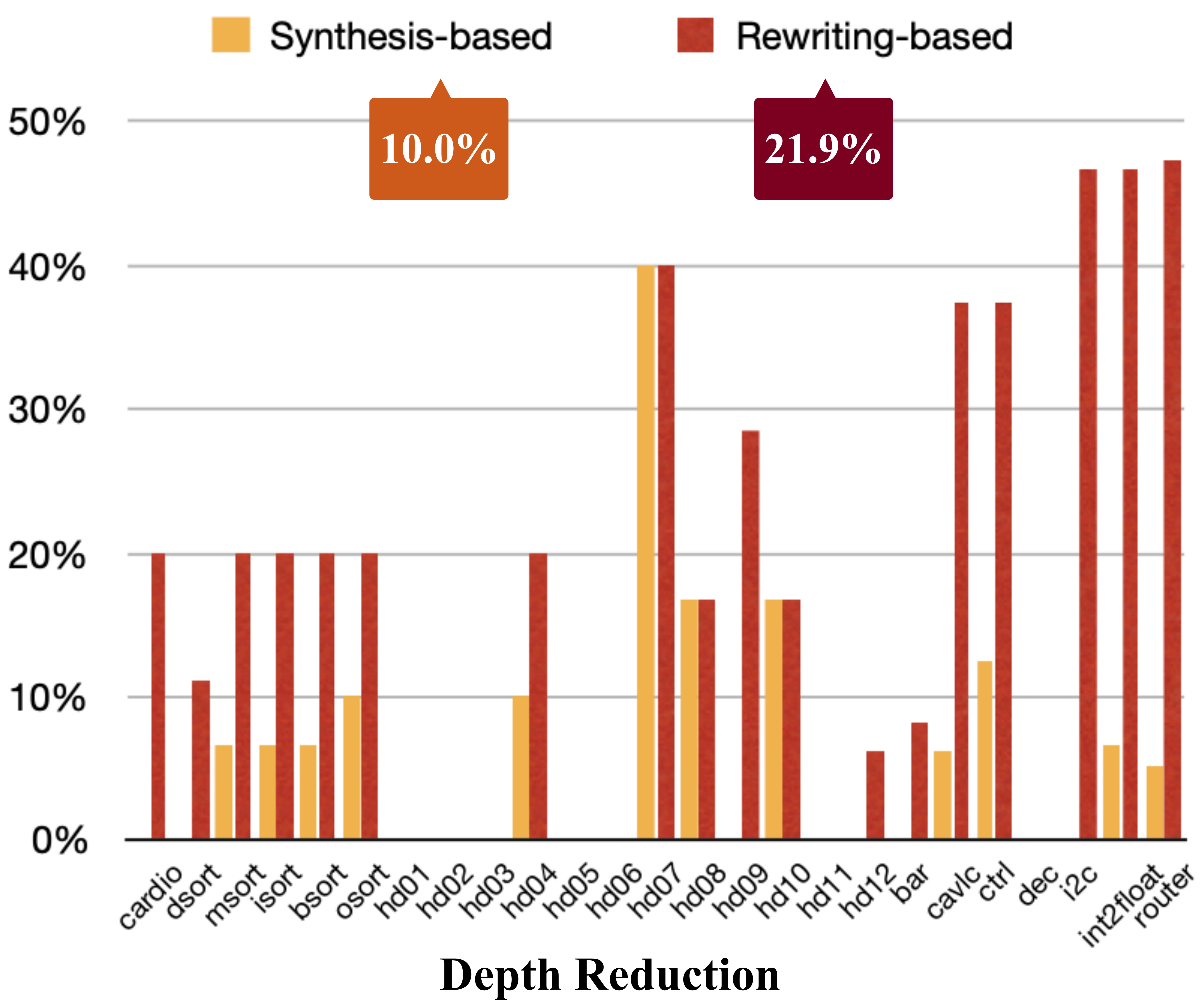
Lobster Performance (3/5)

Efficacy of Reusing Learned Optimization Patterns



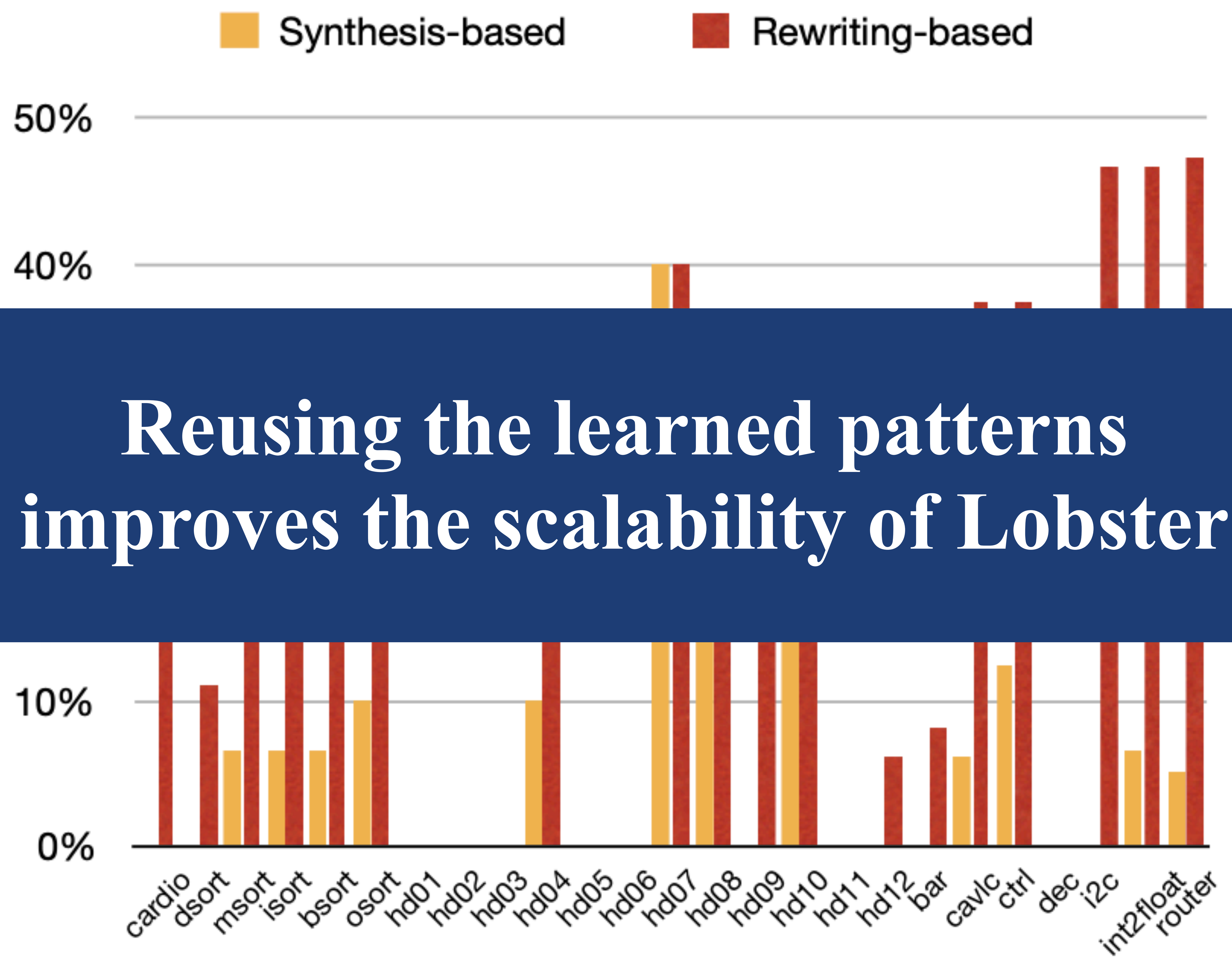
Lobster Performance (3/5)

Efficacy of Reusing Learned Optimization Patterns



Lobster Performance (3/5)

Efficacy of Reusing Learned Optimization Patterns

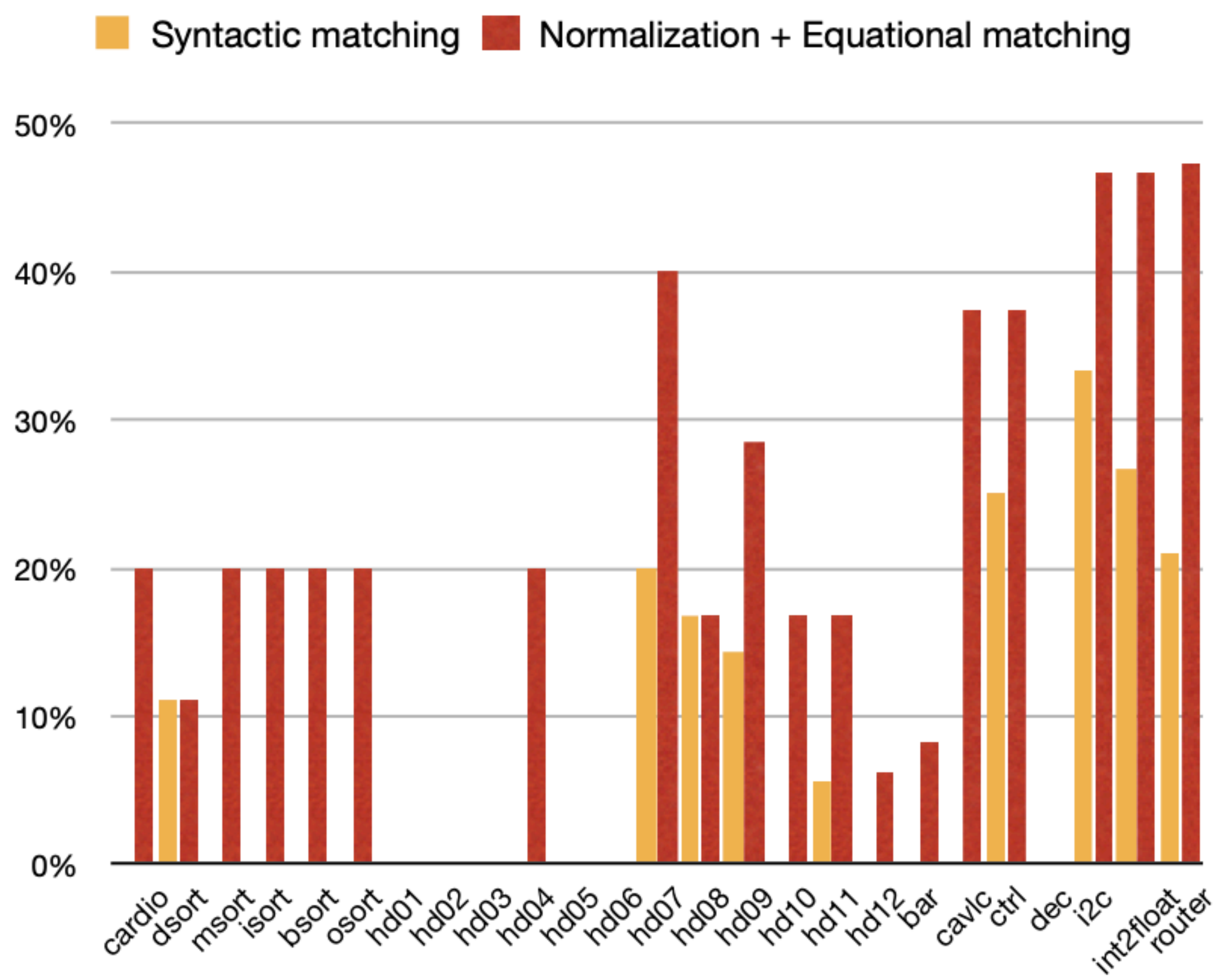


Reusing the learned patterns improves the scalability of Lobster

Depth Reduction

Lobster Performance (4/5)

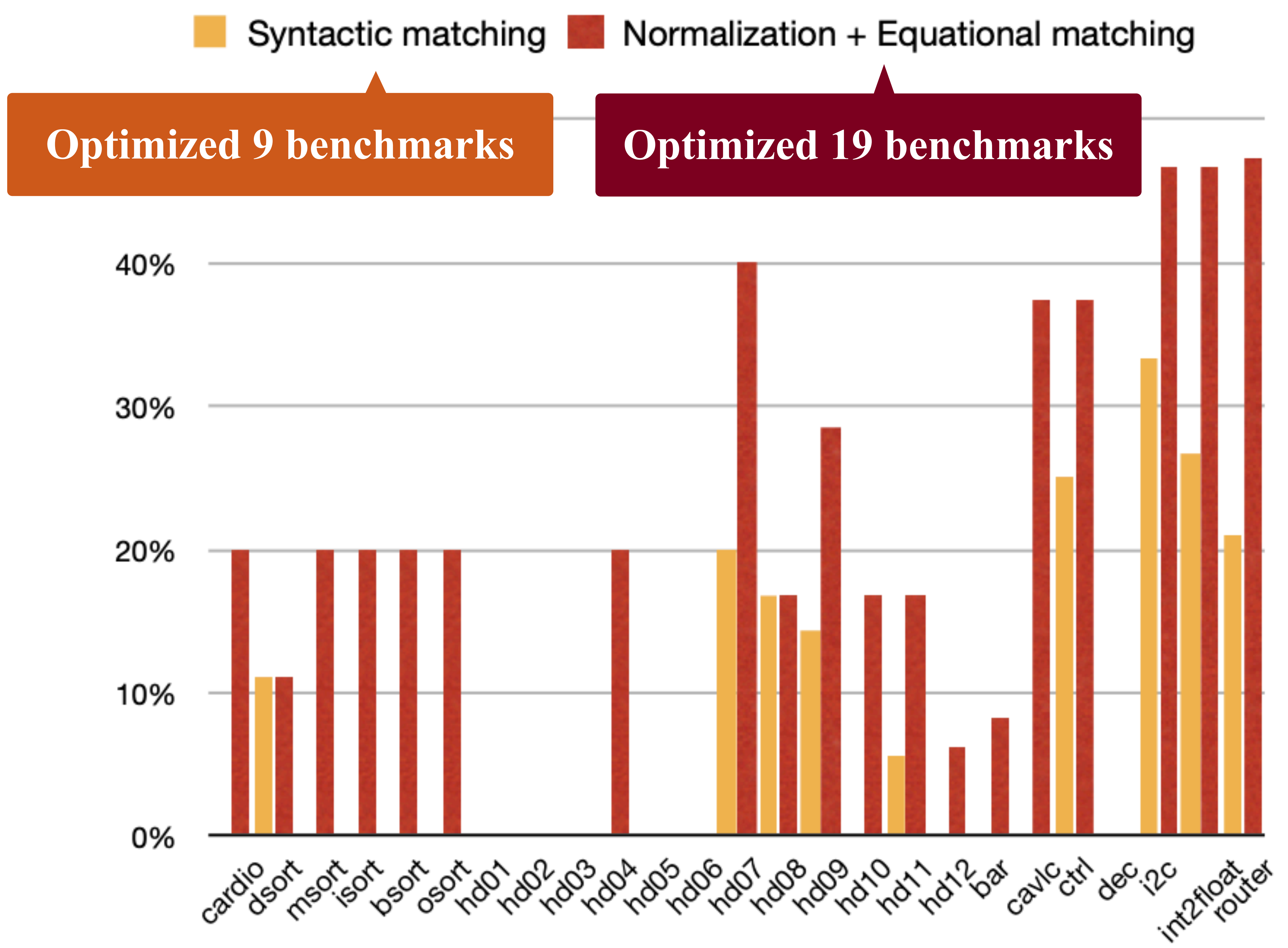
Effectiveness of Equational Term Rewriting



Depth Reduction

Lobster Performance (4/5)

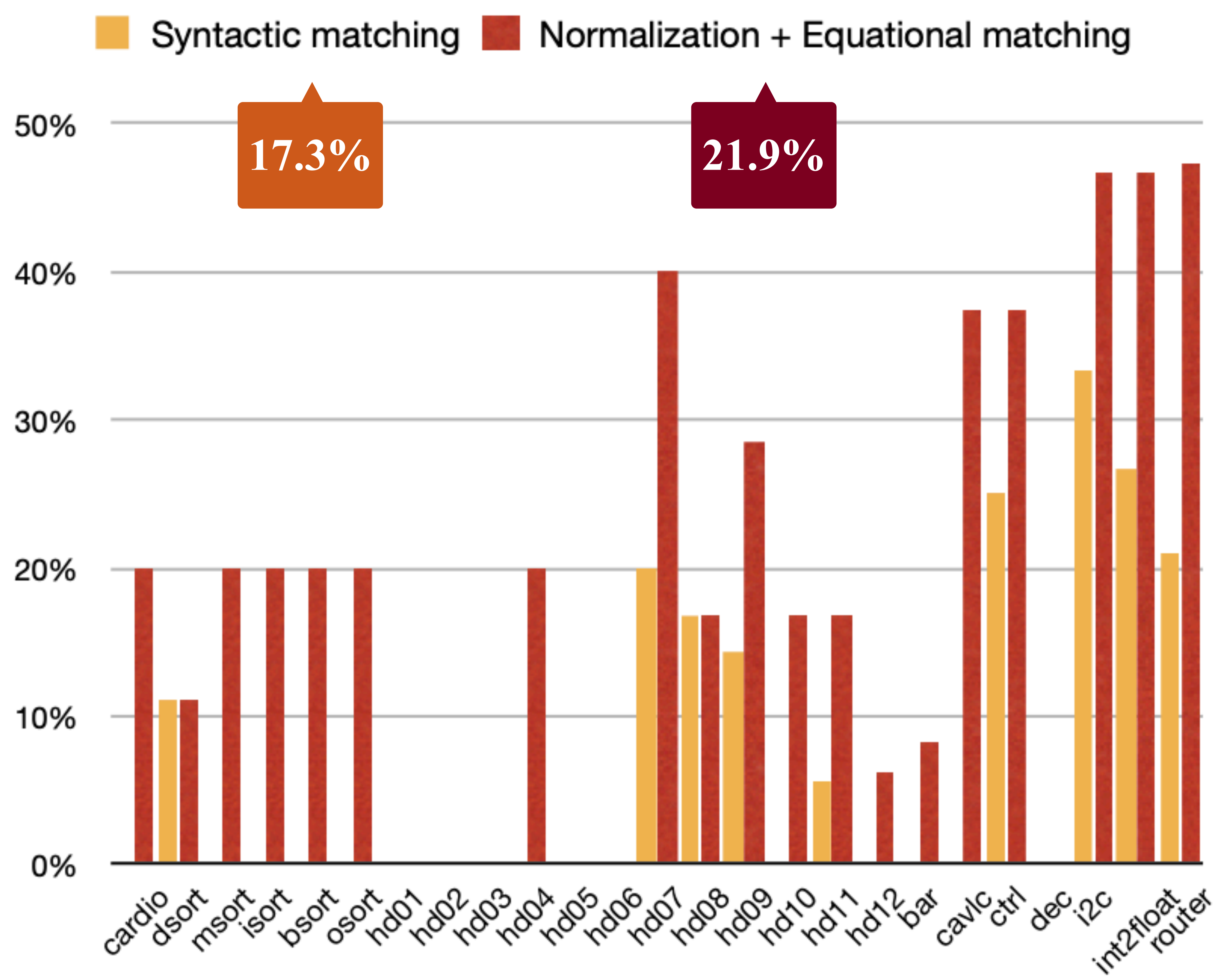
Effectiveness of Equational Term Rewriting



Depth Reduction

Lobster Performance (4/5)

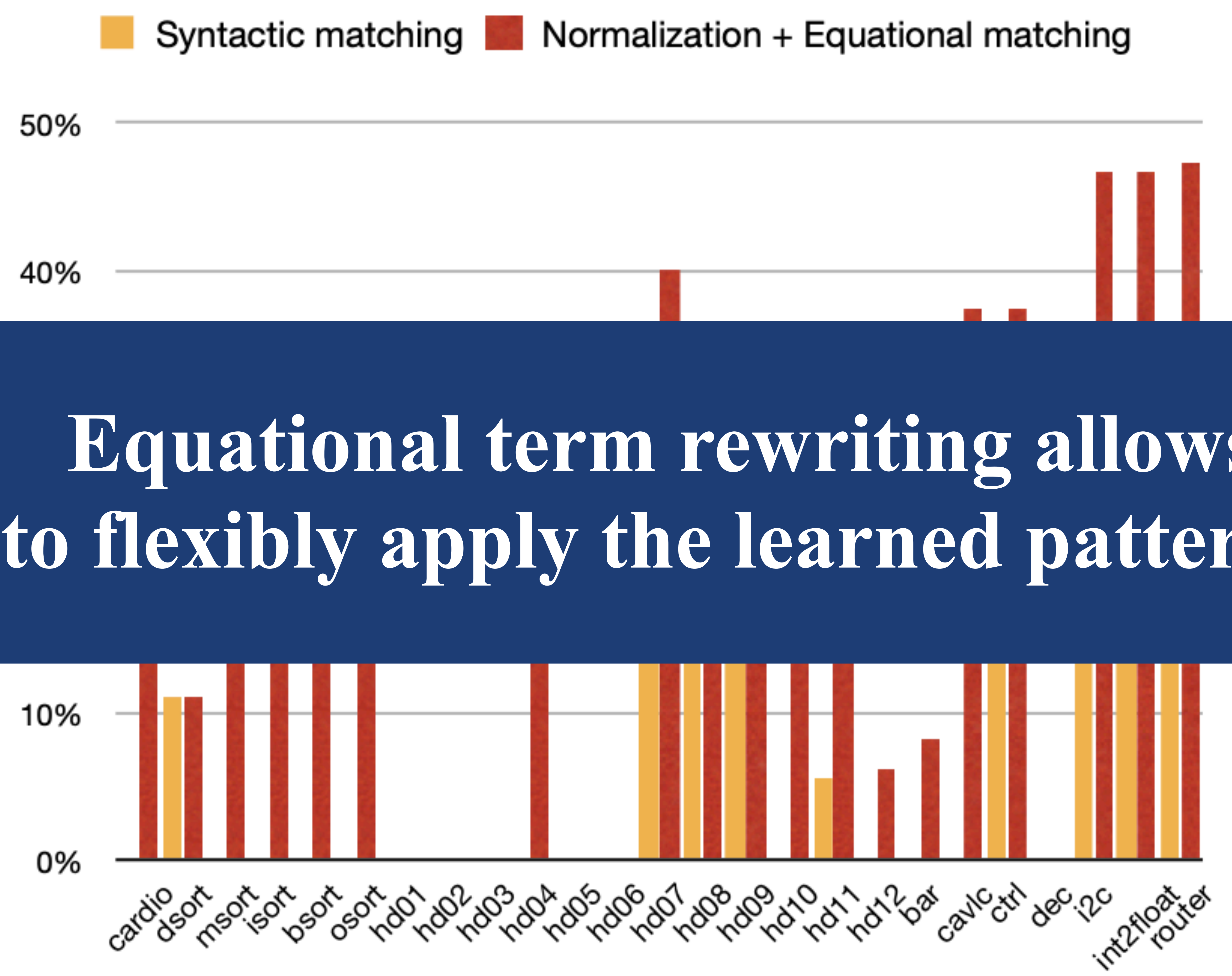
Effectiveness of Equational Term Rewriting



Depth Reduction

Lobster Performance (4/5)

Effectiveness of Equational Term Rewriting

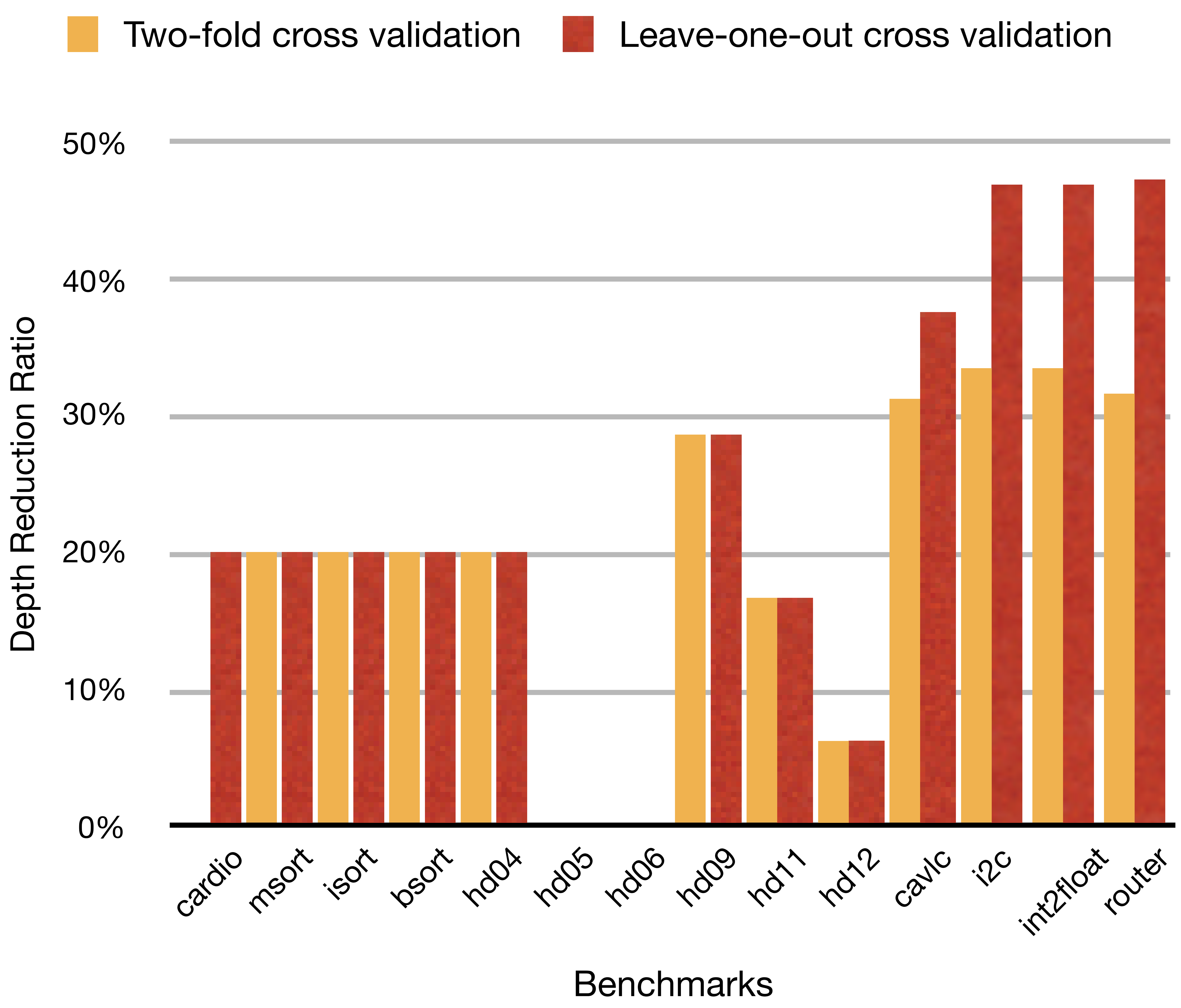


Equational term rewriting allows to flexibly apply the learned patterns

Depth Reduction

Lobster Performance (5/5)

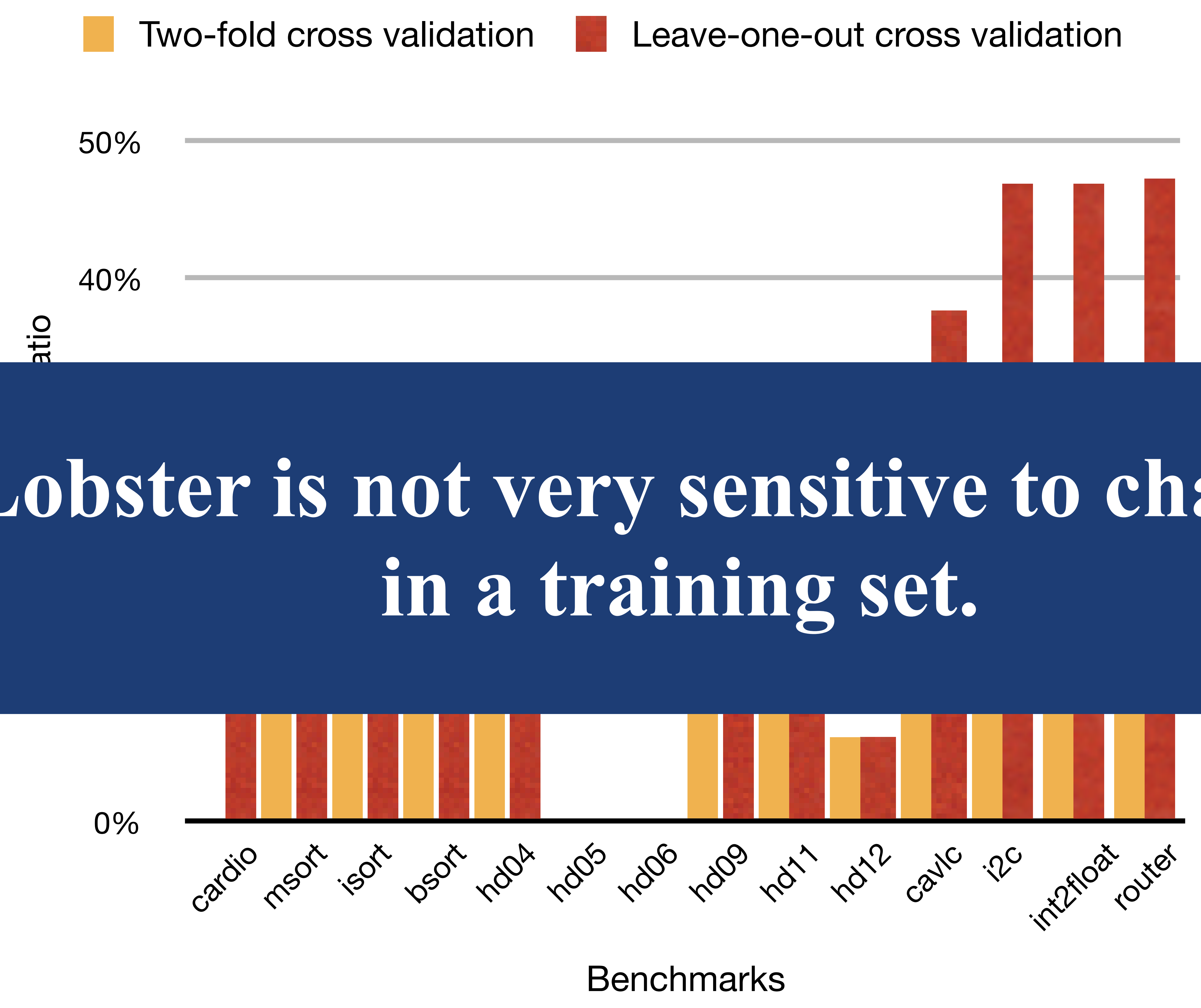
Effectiveness of Equational Term Rewriting



Depth Reduction

Lobster Performance (5/5)

Effectiveness of Equational Term Rewriting



Lobster is not very sensitive to changes in a training set.

Depth Reduction

In the Paper…

- Detailed description of synthesis via localization
- Formalized Equational Term Rewriting
- Detailed description of experiment results

Thank you!